Unit #7—Integration (Antiderivatives)

You are responsible for doing all of the homework and checking your work. If you get stuck, the solutions are worked out at the end of the unit and the odd numbered exercises are also available online through the textbook publisher. If you still have questions on the homework problems after going over the solutions, then come in at lunch by appointment, afterschool, or during intervention as class time will not be devoted to going over the homework.

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Handout

Assignment #4: Trapezoidal Rule
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Assignment #5: Infinite Riemann Sums
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Assignment #9: Second Fundamental Theorem of Calculus
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Assignment #10: Review
Handout

Test

2019-2020
Warm-Up

Give the derivative of each.

1. \( y = x^5 \)
   \( y' = 5x^4 \)

2. \( y = x^{-3} \)
   \( y' = -3x^{-4} \)

3. \( y = \sqrt{x} \)
   \( y = x^{\frac{1}{2}} \)
   \( y' = \frac{1}{2}x^{-\frac{1}{2}} \)

4. \( y = 3 + \ln x \)
   \( y' = \frac{1}{x} \)

5. \( y = e^{3x} \)
   \( y' = 3e^{3x} \)

6. \( y = 3^{5x} \)
   \( y' = 3^{5x} \ln(3) \cdot 5 \)

7. \( y = 5x^2 + 3 \)
   \( y' = 10x \)

8. \( y = 5x^2 - 9 \)
   \( y' = 10x \)

9. \( y = 5x^2 + 143.2 \)
   \( y' = 10x \)

10. Given the graph of \( f'(x) \) and \( f(1) = 7 \).

   A) Identify the \( x \)-coordinates of any maximum values of \( f(x) \).
   Justify your answer.
   
   max at \( x = -2 \) b/c \( f' \) goes from pos to neg

   B) Identify the \( x \)-coordinates of any minimum values of \( f(x) \).
   Justify your answer.
   
   min at \( x = 4 \) b/c \( f' \) goes from neg to pos

   C) Identify the \( x \)-coordinates of any points of inflection of \( f(x) \). Justify your answer.
   
   \( f \) has a poi at \( x = 1 \) b/c \( f' \) goes from dec to inc

   D) Give the equation of the tangent line of \( f \) at \( x = 1 \).
   
   Point \( (1, 7) \)
   Slope \(-4\)
   \( y - 7 = -4(x - 1) \)
AP Calculus AB

Topic: Antiderivatives and Indefinite Integration

Goal: Be able to find the antiderivative of a simple function.

Definition Antiderivative:
A function F is an antiderivative of f on an open interval I if
\[ F'(x) = f(x) \quad \forall x \in I. \]

Antidifferentiation (or indefinite integration) is an operation.

\[ \int f(x) \, dx = F(x) + C \]

Integrand → Constant of Integration → Variable of Integration

Indefinite Integral is a synonym for antiderivative.

Every rule for derivatives has a companion rule for integrals.

**What Functions Derivative equals the integrand?**

\[ \int 2x \, dx = x^2 + C \]

\[ \int 4x^3 \, dx = x^4 + C \]

\[ \int x^3 + x^2 - 3x + 4 = \frac{1}{4} x^4 + \frac{1}{3} x^3 - \frac{1}{2} x^2 + 4x + C \]

\[ \int \frac{1}{x^5} \, dx = \int x^{-5} \, dx \]

\[ \int x^5 \, dx = \frac{1}{6} x^6 + C \]

\[ \int x^{29} \, dx = \frac{1}{30} x^{30} + C \]

\[ \int \tan x \, dx = \sec^2 x \rightarrow \int \sec^2 x \, dx = \tan x + C \]

**How it Works**

If \( f(x) = 7x^3 \), then \( f'(x) = 21x^2 \)

If \( f'(x) = 10x^4 \), then \( f(x) = 2x^5 + C \)

\( \text{some constant} \)

**Why it Works**

\( y' = 4x \) another way to write it is \( \frac{dy}{dx} = 4x \)

\[ \frac{dy}{dx} = \int 4x \, dx \]

Separate dy and dx.

\[ \int dy = \int 4x \, dx \]

Integrate both sides (antidifferentiate).

\[ y = 2x^2 + C \]

\[ \frac{d}{dx} [y = 2x^2 + C] \]

\[ \frac{dy}{dx} = 4x \]

Taking the derivative.

\[ \frac{d}{dx} \tan x = \sec^2 x \rightarrow \int \sec^2 x \, dx = \tan x + C \]

**To check each, take the derivative.**
AP Calculus AB

\[ \int \frac{1}{x^7} \, dx = -\frac{1}{6} x^{-6} + C \]

\[ \int \frac{1}{x^{11}} \, dx = -\frac{1}{10} x^{-10} + C \]

\[ \int \frac{1}{x} \, dx = \ln|x| + C \quad \text{\(x\) cannot be neg} \]

\[ \int \frac{x^3}{\sqrt{x}} \, dx = \int x^{\frac{3}{2}} - 3x^{-\frac{1}{2}} \, dx = \frac{2}{3} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + C \]

\[ \int 5e^x \, dx = 5e^x + C \]

\[ \int \sqrt{x} \, dx = \frac{2}{3} x^{\frac{3}{2}} + C \quad \text{reciprocals} \]

\[ \int x^8 \, dx = \frac{8}{9} x^{\frac{9}{3}} + C \]

\[ \int x^{4-2x} \, dx = \int x^3 - 2 \, dx = \frac{1}{4} x^4 - 2x + C \]

\[ \int x^{-3} \, dx = \int -\frac{3}{x^3} \, dx = \int x^{-\frac{3}{2}} \, dx = \frac{3}{5} x^{\frac{5}{3}} - \frac{3}{2} x^{\frac{2}{3}} + C \]

\[ \int \frac{x^2+5x+6}{x+3} \, dx = \int \frac{(x+3)(x+2)}{x+3} \, dx = \int x + 2 \, dx = \frac{1}{2} x^2 + 2x + C \]

Trigonometric Antiderivatives

\[ \int \sin x \, dx = -\cos x + C \]

\[ \int \sec^2 x \, dx = \tan x + C \]

\[ \int \sec x \tan x \, dx = \sec x + C \]

\[ \int \cos x \, dx = \sin x + C \]

\[ \int \csc^2 x \, dx = -\cot x + C \]

\[ \int \csc x \cot x \, dx = -\csc x + C \]

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AP Calculus AB

Warm-Up

Lesson 2

Unit #7: Integration

Give the anti-derivative.

1. If \( \frac{dy}{dx} = x^2 + 3 \), then
   \[ y = \frac{1}{3} x^3 + 3x + C \]

   How would the answer change if I told you that when \( x = 1, \ y = 2 \)?
   \[ y = \frac{1}{3} x^3 + 3x + C \]
   \[ 2 = \frac{1}{3} (1)^3 + 3(1) + C \]
   \[ 2 = \frac{1}{3} + 3 + C \]
   \[ -\frac{4}{3} = C \]

   \[ C = -\frac{4}{3} \]

   \[ y = \frac{1}{3} x^3 + 3x - \frac{4}{3} \]

   This is a specific or particular solution.

2. If \( f''(x) < 0 \) on the interval \((1, 4)\), \( f(2) = 7 \), and \( f'(2) = \frac{1}{3} \), then

   a) What is the equation of the tangent line at \( x = 2 \)?

      **Point**
      \( (2, 7) \)

      **Slope** = \( \frac{1}{3} \)

      \[ y-7 = \frac{1}{3} (x-2) \]

   b) Use the tangent line to approximate \( f(2.3) \).

      \[ y-7 = \frac{1}{3} (2.3 - 2) \]
      \[ y-7 = \frac{1}{3} \cdot \frac{3}{10} \]
      \[ y = \frac{10}{3} \]

   c) Is the approximation greater than or less than the actual value? Explain.

      The approximation is greater than the actual value because \( f'' < 0 \) \( \Rightarrow \) \( f \) is concave down.

      \[ 7 \]
Initial Value Problems

Solving first order differential equations. You want to solve for \( y \).

\[ y' = f(x) \]

\( \frac{dy}{dx} = f(x) \)

\( dy = f(x) \, dx \)

Integrating both sides gives:

\[ \int dy = \int f(x) \, dx \]

\[ y = F(x) + c \]

(antiderivative of \( f(x) \))

Solve: \( y' = \frac{1}{x^2}, x > 0 \) and \( F(1) = 0 \). (Initial value problem)

\[ \frac{dy}{dx} = x^{-2} \]

\[ dy = x^{-2} \, dx \]

\[ \int dy = \int x^{-2} \, dx \]

\[ y = -1 \cdot x^{-1} + C \]

\[ y = -\frac{1}{x} + C \]

\[ 0 = -\frac{1}{1} + C \]

\[ 1 = C \]

\[ y = -\frac{1}{x} + 1 \]

Particular Solution
Solve: \( \frac{dy}{dx} = 9x^2 - 4x + 5 \quad y(-1) = 0 \)

\[
dy = 9x^2 - 4x + 5 \, dx
\]

\[
5 \, dy = 5 \, 9x^2 - 4x + 5 \, dx
\]

\[
y = 3x^3 - 2x^2 + 5x + C
\]

\[
o = 3(-1)^3 - 2(-1)^2 + 5(-1) + C
\]

\[
o = -3 - 2 - 5 + C
\]

\[
o = -10 + C
\]

\[
c = 10
\]

\[
y = 3x^3 - 2x^2 + 5x + 10
\]

Solve: \( \frac{dy}{dx} = 3x^2 + 2x^2 + 7x \quad y(1) = 1 \)

\[
dy = 3x^2 + 2x^2 + 7x \, dx
\]

\[
5 \, dy = 5 \, 3x^2 + 2x^2 + 7x \, dx
\]

\[
y = \frac{1}{4} \cdot 3x^4 + \frac{1}{3} \cdot 2x^3 + \frac{1}{2} \cdot 7x^2 + C
\]

\[
\left[ 1 = \frac{1}{4} \cdot 3 + \frac{1}{3} \cdot 2 + \frac{1}{2} \cdot 7 + C \right]_{12}
\]

\[
12 = 9 + 8 + 42 + 12C
\]

\[
12 = 59 + 12C
\]

\[
-47 = 12C
\]

\[
c = -\frac{47}{12}
\]

\[
y = \frac{3}{4}x^4 + \frac{2}{3}x^3 + \frac{7}{2}x^2 - \frac{47}{12}
\]
Lesson 3

Topic: Approximate the Area Under a Curve

Goal: Approximate the area under a curve using LRAM, MRAM, RRAM (Reimann Sums).

Approximating the Area Under a Curve

Rectangular Approximation Methods

**LRAM:** Left Rectangular Approximation Method

\[
A = (c-a) f(a) + (b-c) f(c)
\]

**RRAM:** Right Rectangular Approximation Method

\[
A = (c-a) f(c) + (b-c) f(b)
\]

**MRAM:** Midpoint Rectangular Approximation

Use middle height

\[
A = (c-a) f\left(\frac{a+c}{2}\right) + (b-c) f\left(\frac{c+b}{2}\right)
\]

All bases are the same
Use the table below to approximate the area under the curve using \(3\) equal intervals and LRAM, RRAM, and MRAM.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>22</td>
</tr>
</tbody>
</table>

**LRAM**

\[
b \cdot h = A
\]

\[
[0,4] \quad 4 \cdot 4 = 16
\]

\[
[4,8] \quad 4 \cdot 2 = 8
\]

\[
[8,12] \quad 4 \cdot 8 = 32
\]

**Intervals**

\[
\frac{12 - 0}{3} = 4\quad \text{wide}
\]

**Height of left side of rect.**

\[
A = 56 \text{ un}^2
\]

**Right RRAM**

\[
6 \cdot h = A
\]

\[
[0,4] \quad 4 \cdot 2 = 8
\]

\[
[4,8] \quad 4 \cdot 8 = 32
\]

\[
[8,12] \quad 4 \cdot 22 = 88
\]

\[
A = 128 \text{ un}^2
\]

**Midpoint**

\[
[0,4] \quad 4 \cdot 2 = 8
\]

\[
[4,8] \quad 4 \cdot 4 = 16
\]

\[
[8,12] \quad 4 \cdot 14 = 56
\]

\[
A = 80 \text{ un}^2
\]
Find the area of the curve bounded by \( y = 3x^2 \) and the x-axis from \( x = 0 \) to \( x = 2 \) using \( n = 4 \) (note: this denotes the number of rectangles.)

\[
\begin{align*}
[0, \frac{1}{2}] & \quad \frac{1}{2} \cdot f(0) = \frac{1}{2} \cdot 0 = 0 \\
[\frac{1}{2}, 1] & \quad \frac{1}{2} \cdot f(\frac{1}{2}) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \\
[1, \frac{3}{2}] & \quad \frac{1}{2} \cdot f(1) = \frac{1}{2} \cdot 3 = \frac{3}{2} \\
[\frac{3}{2}, 2] & \quad \frac{1}{2} \cdot f(\frac{3}{2}) = \frac{1}{2} \cdot \frac{27}{4} = \frac{27}{8} \\
\end{align*}
\]

\[
A = \frac{42}{8}
\]

\[
\begin{align*}
[0, \frac{1}{2}] & \quad \frac{1}{2} \cdot f(\frac{1}{2}) = \frac{1}{2} \cdot \frac{3}{16} = \frac{3}{32} \\
[\frac{1}{2}, 1] & \quad \frac{1}{2} \cdot f(1) = \frac{1}{2} \cdot \frac{27}{16} = \frac{27}{32} \\
[1, \frac{3}{2}] & \quad \frac{1}{2} \cdot f(\frac{3}{2}) = \frac{1}{2} \cdot \frac{75}{16} = \frac{75}{32} \\
[\frac{3}{2}, 2] & \quad \frac{1}{2} \cdot f(2) = \frac{1}{2} \cdot 147 = \frac{147}{32} \\
\end{align*}
\]

\[
A = \frac{252}{32}
\]

Midpoints:
\[
\begin{align*}
0 + \frac{1}{2} & = \frac{1}{4} \\
\frac{1}{2} + 1 & = \frac{3}{4} \\
1 + \frac{3}{2} & = \frac{5}{4}
\end{align*}
\]

\[
2019-2020
\]
Approximate the area of the curve bounded by \( y = x^2 \) and the x-axis from \( x = -2 \) to \( x = 2 \) using \( n = 4 \) using Riemann Sums.

<table>
<thead>
<tr>
<th>Interval</th>
<th>LRAM</th>
<th>RRAM</th>
<th>MRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>([-2, -1])</td>
<td>(1) ( f(-2) = -8 )</td>
<td>(1) ( f(-1) = -1 )</td>
<td>(1) ( f(-\frac{3}{2}) = -\frac{27}{8} )</td>
</tr>
<tr>
<td>([-1, 0])</td>
<td>(1) ( f(-1) = -1 )</td>
<td>(1) ( f(0) = 0 )</td>
<td>(1) ( f(-\frac{1}{2}) = -\frac{1}{8} )</td>
</tr>
<tr>
<td>([0, 1])</td>
<td>(1) ( f(0) = 0 )</td>
<td>(1) ( f(1) = 1 )</td>
<td>(1) ( f(\frac{1}{2}) = \frac{1}{8} )</td>
</tr>
<tr>
<td>([1, 2])</td>
<td>(1) ( f(1) = 1 )</td>
<td>(1) ( f(2) = 8 )</td>
<td>(1) ( f(\frac{3}{2}) = \frac{27}{8} )</td>
</tr>
</tbody>
</table>

\[ A = 10 \quad A = \frac{56}{8} \]

Let the table of values be for a continuous function. Use the table below to approximate \( \int_{0}^{42} f(x) \, dx \) using LRAM and RRAM using 6 intervals.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>LRAM</th>
<th>RRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>16</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>38</td>
<td>8</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>42</td>
<td>19</td>
<td>6</td>
<td>19</td>
</tr>
</tbody>
</table>

\[ A = 481 \quad A = 479 \]

Why did we not do the table by MRAM? b/c we don't know midpoint values.
Topic: Trapezoidal Rule

Goal: Approximate the area under a curve using the trapezoidal rule.
Formula for the area of a trapezoid is
\[ A = \frac{1}{2} h(b_1 + b_2) \]

Use the trapezoid rule with four trapezoids to approximate the area under the curve given by the points in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>4</th>
<th>6</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>5</td>
<td>12</td>
<td>15</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

\[ T_1 = \frac{1}{2} (5 + 12) \cdot 3 = \frac{51}{2} \]
\[ T_2 = \frac{1}{2} (12 + 15) \cdot 2 = 27 \]
\[ T_3 = \frac{1}{2} (15 + 20) \cdot 4 = 50 \]
\[ T_4 = \frac{1}{2} (20 + 22) \cdot 5 = 105 \]
\[ \int_1^{15} f(x) \, dx \approx \frac{51}{2} + 27 + 50 + 105 \]

Use the trapezoid rule with four trapezoids to approximate the area under the curve.

\[ \int_0^4 x^2 + 1 \, dx \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

\[ T_1 = \frac{1}{2} (1 + 2) \cdot 1 = \frac{3}{2} \]
\[ T_2 = \frac{1}{2} (2 + 5) \cdot 1 = \frac{7}{2} \]
\[ T_3 = \frac{1}{2} (5 + 10) \cdot 1 = \frac{15}{2} \]
\[ T_4 = \frac{1}{2} (10 + 17) \cdot 1 = \frac{27}{2} \]
\[ \int_0^4 x^2 + 1 \, dx \approx \frac{52}{2} \]
Use the data in the table to approximate the area under the curve on the interval [1, 16], using Left Riemann Sums and four subintervals.

<table>
<thead>
<tr>
<th>Interval</th>
<th>b</th>
<th>h</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1, 3]</td>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>[3, 8]</td>
<td>5</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>[8, 10]</td>
<td>2</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>[10, 16]</td>
<td>6</td>
<td>15</td>
<td>90</td>
</tr>
</tbody>
</table>

LRAM

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>3</th>
<th>8</th>
<th>10</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>6</td>
<td>2</td>
<td>12</td>
<td>15</td>
<td>2</td>
</tr>
</tbody>
</table>

Trap

\[ T_1 = \frac{1}{2}(6+2)2 = 8 \]
\[ T_2 = \frac{1}{2}(2+12)5 = 35 \]
\[ T_3 = \frac{1}{2}(12+15)2 = 27 \]
\[ T_4 = \frac{1}{2}(15+2)6 = 51 \]

\[ A = 136 \text{ un}^2 \]

**Assignment #2:** Page 461: 1, 11 Ignore the directions and approximate the area bounded by the curve and the x-axis using Left Riemann Sums, Right Riemann Sums, Midpoint Sums, and the Trapezoidal Method. Let the number of intervals be what is given in the book.
Lesson 5

Topic: Infinite Riemann Sums

Goal: To find the area under a curve using the limit of an infinite Riemann sum.

Approximate the area of the curve bounded by \( y = x^2 + 1 \) and the x-axis from \( x = 1 \) to \( x = 3 \).

If right-endpoint approximations are used with \( n = 6 \), what are the values of \( x \)? Label them on the graph.

How would you find the height of each rectangle? Do it.

\[
\begin{align*}
\gamma_1 &= f\left(\frac{1}{3}\right) = \frac{16}{9} + 1 = \frac{25}{9} \\
\gamma_2 &= f\left(\frac{2}{3}\right) = \frac{25}{9} + 1 = \frac{34}{9} \\
\gamma_3 &= f(2) = 4 + 1 = 5 \\
\gamma_4 &= f\left(\frac{7}{3}\right) = \frac{49}{9} + 1 = \frac{58}{9} \\
\gamma_5 &= f\left(\frac{8}{3}\right) = \frac{64}{9} + 1 = \frac{73}{9} \\
\gamma_6 &= f(3) = 9 + 1 = 10
\end{align*}
\]

Since all bases are the same

\[ A = \frac{1}{3} \left( \frac{25}{9} + \frac{34}{9} + 5 + \frac{58}{9} + \frac{73}{9} + 10 \right) \]

How would you estimate the total area under the function on this interval? Do it.
Approximate the area of the curve bounded by \( y = x^2 + 1 \) and the x-axis from \( x = 1 \) to \( x = 3 \).

If left–endpoint approximations are used with \( n = 6 \), what are the values of \( x \)? How would you find the height of each rectangle? How would you find the area of each rectangle? Do it.

\[
\begin{align*}
\gamma_1 & : f(1) = 2 \\
\gamma_2 & : f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^2 + 1 = \frac{25}{9} \\
\gamma_3 & : f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^2 + 1 = \frac{34}{9} \\
\gamma_4 & : f\left(\frac{7}{3}\right) = \left(\frac{7}{3}\right)^2 + 1 = \frac{58}{9} \\
\gamma_5 & : f\left(\frac{8}{3}\right) = \left(\frac{8}{3}\right)^2 + 1 = \frac{73}{9} \\
\gamma_6 & : f(3) = 3^2 + 1 = 10
\end{align*}
\]

\[
\frac{1}{3} \left( 2 + \frac{25}{9} + \frac{34}{9} + 10 + \frac{58}{9} + \frac{73}{9} \right) = 12.037
\]

Now, what about right–endpoint approximations with \( n = 50 \)? Write the first few two terms, the last two terms, and then the sigma notation for the total approximation.

\[
\frac{\text{length of interval}}{\text{# of intervals}} = \frac{2}{50} = \frac{1}{25} \quad \text{base of each rect}
\]

\[
\begin{align*}
\gamma_1 & : f(1 + \frac{1}{25}) \quad A = \frac{1}{25} \cdot f(1 + \frac{1}{25}) \\
\gamma_2 & : f(1 + \frac{2}{25}) \quad A = \frac{1}{25} \cdot f(1 + \frac{2}{25}) \\
\gamma_3 & : f(1 + \frac{3}{25}) \quad A = \frac{1}{25} \cdot f(1 + \frac{3}{25}) \\
& \vdots \\
\gamma_{49} & : f(1 + \frac{49}{25}) \quad A = \frac{1}{25} \cdot f(1 + \frac{49}{25}) \\
\gamma_{50} & : f(1 + \frac{50}{25}) \quad A = \frac{1}{25} \cdot f(1 + \frac{50}{25}) \\
\end{align*}
\]

\[
A = \sum_{n=1}^{50} \frac{1}{25} \cdot f(1 + \frac{n}{25}) = 10.827
\]
Approximate the area of the curve bounded by $y = x^2 + 1$ and the x-axis from $x = 1$ to $x = 3$.

YOU TRY:
What about left-endpoint approximations with $n = 100$? Write the first few terms, the last two terms, and then the sigma notation for the total approximation.

\[
\sum_{n=1}^{100} \frac{1}{50} \left[ (1 + \frac{n-1}{50})^2 + 1 \right] = 10.587
\]

If $f(x)$ is continuous on $[a, b]$, then the endpoint and midpoint approximations approach one and the same limit as $N \to \infty$. In other words, there is a value $L$, such that

\[
\lim_{N \to \infty} R_N = \lim_{N \to \infty} L_N = \lim_{N \to \infty} M_N = L.
\]

If $f(x) \geq 0$, we define the area under the graph over $[a, b]$ to be $L$.

1. Let $A$ be the area under the graph of $f(x)$, how you would set up an infinite Riemann sum to find $A$.

Assignment #5: Handout

2019-2020
Find the area of each shape. **No calculator.**

- Triangle: \[ A = \frac{1}{2} (12)(3) \]
  \[ A = 18 \text{ ft}^2 \]

- Circle: \[ A = \pi \cdot 8^2 \]
  \[ A = 64\pi \text{ ft}^2 \]

- Trapezoid: \[ A = \frac{1}{2} (5 + 24)(10) \]
  \[ A = 140 \text{ ft}^2 \]

- Triangle: \[ A = \frac{\sqrt{3}}{4} s^2 \]
  \[ A = \frac{\sqrt{3}}{4} \times 7^2 \]
  \[ A = \frac{49\sqrt{3}}{4} \text{ ft}^2 \]
Topic: Evaluating Integrals using Geometry

Goal: Be able to find the area under a curve using geometry.

An accurate graph will make it a lot easier to setup and evaluate the integrals needed.

$$\int_{-1}^{5} |x| \, dx$$

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x > 0 \end{cases}$$

$$\frac{1}{2}(1)(1) + \frac{1}{2}(5)(5)$$

$$\frac{1}{2} + \frac{25}{2}$$

$$= \frac{26}{2}$$

$$\int_{0}^{2} [2x - 3] \, dx$$

vertex

$$2x - 3 = 0$$

$$x = \frac{3}{2}$$

$$\frac{1}{2}(\frac{3}{2})(3) + \frac{1}{2}(\frac{1}{2})(1)$$

$$\frac{9}{4} + \frac{1}{4}$$

$$= \frac{10}{4}$$

$$\int_{-2}^{3} 5x \, dx$$

$$-\frac{1}{2}(2)(10) + \frac{1}{2}(3)(15)$$

$$-10 + \frac{45}{2}$$

$$-\frac{20}{2} + \frac{45}{2}$$

Area under x-axis

15 Negative

2018-2019
\[ \int_0^5 3x - 6 \, dx \]

\[ \int_2^5 \sqrt{4-x^2} \, dx \]

\[ \int_{-3}^3 \sqrt{9-x^2} \, dx \]

\[ \frac{1}{2} \pi (2)^2 \]

\[ 2 \pi \]

\[ -\frac{9}{2} \pi \]

\[ \frac{15}{2} \]

under x-axis above

\[ -\frac{1}{2} (2)(6) + \frac{1}{2} (3)(9) \]

\[ -6 + \frac{27}{2} \]

\[ -\frac{12}{2} + \frac{27}{2} \]

\[ \frac{15}{2} \]
Lesson 7

Topic: Definite Integral

Goal: Evaluate an Integral using the first Fundamental Theorem of Calculus.

Indefinite Integral
\[ \int f(x) \, dx = F(x) + C \]

General Antiderivative

Definite Integral
\[ \int_a^b f(x) \, dx = F(b) - F(a) \]

Area between function and x-axis

There are limits of integration: \( a \) and \( b \).
\( a \) is the lower limit and \( b \) is the upper limit.

The graph at the right is \( f(x) \).

A. Write a piecewise function for \( f(x) \).

\[
f(x) = \begin{cases} 
2x & , \quad x < 3 \\
-\frac{5}{2}x + \frac{25}{2} & , \quad x \geq 3 
\end{cases}
\]

B. There are three triangles formed by the graph and the x-axis on the interval \([-3, 7]\).

Find the area of each triangle.

\begin{align*}
\text{Area below the x-axis is negative}
\end{align*}

C. Use the areas from part B to evaluate:

1) \[ \int_0^5 f(x) \, dx = \frac{25}{2} \]

2) \[ \int_{-3}^0 f(x) \, dx = -\frac{15}{2} \]

3) \[ \int_5^7 f(x) \, dx = -5 \]

4) \[ \int_{-3}^5 f(x) \, dx = -\frac{15}{2} + \frac{25}{2} = 5 \]

5) \[ \int_{-3}^7 f(x) \, dx = -\frac{15}{2} + \frac{25}{2} - \frac{10}{2} = 0 \]

6) \[ \int_0^7 f(x) \, dx = \frac{25}{2} - 5 = \frac{10}{2} \]

7) \[ \int_0^1 f(x) \, dx = \frac{1}{2}(1) \cdot \frac{5}{3} = \frac{5}{6} \]

8) \[ \int_1^1 f(x) \, dx = \frac{1}{2} \cdot 0 \cdot 0 = 0 \]

2019-2020
First Fundamental Theorem of Calculus

If \( f(x) \) is continuous on \([a, b]\) and \( F(x) \) is the antiderivative of \( f(x) \), then

\[
\int_a^b f(x) \, dx = F(b) - F(a)
\]

1) \( f(x) = \begin{cases} \frac{5}{3}x, & x < 3 \\ -\frac{5}{2}x + \frac{25}{2}, & x \geq 3 \end{cases} \)

\[
\int_{-3}^{3} f(x) \, dx = \left[ \frac{5}{3} \cdot \frac{5}{3} x^2 \right]_{-3}^{0} = \frac{5}{9} \cdot 0^2 - \frac{5}{9} \cdot (-3)^2 = -\frac{15}{6} = -\frac{15}{2}
\]

Switch limits

\[
\int_{-3}^{0} f(x) \, dx = \left[ \frac{5}{3} \cdot \frac{5}{3} x^2 \right]_{0}^{-3} = \frac{5}{9} (-3)^2 - \frac{5}{9} \cdot 0^2 = \frac{15}{6} = \frac{15}{2}
\]

2) \( f(x) = \begin{cases} -\frac{5}{2}x + \frac{25}{2}, & x \geq 5 \\ \frac{5}{3}x, & x < 5 \end{cases} \)

\[
\int_{5}^{7} f(x) \, dx = \left[ \frac{5}{3} \cdot \frac{5}{3} x^2 \right]_{5}^{7} = \frac{5}{9} \cdot 7^2 + \frac{25}{2} \cdot 7 - \left( -\frac{5}{4} \cdot 5^2 + \frac{25}{2} \cdot 5 \right)
\]

\[= \frac{245}{4} + \frac{175}{2} - \left( -\frac{125}{4} + \frac{125}{2} \right) = \frac{245}{4} + \frac{350}{4} + \frac{125}{4} - \frac{250}{4} = \frac{20}{4} = -5 \text{ see } \Delta 3 \text{ easier to do geometrically}
\]

\[
\int_{5}^{7} f(x) \, dx = -\int_{5}^{7} f(x) \, dx = 5
\]

3) \( f(x) = \begin{cases} \frac{5}{3}x, & x < 1 \\ \frac{5}{6}, & x \geq 1 \end{cases} \)

\[
\int_{0}^{1} f(x) \, dx = \left[ \frac{5}{3} \cdot \frac{5}{3} x^2 \right]_{0}^{1} = \frac{5}{6} \cdot 1^2 - \frac{5}{6} \cdot 0 = \frac{5}{6}
\]

see #7 previous page

4) \( f(x) = \begin{cases} \frac{5}{3}x, & x < 1 \\ \frac{5}{6}, & x \geq 1 \end{cases} \)

\[
\int_{1}^{4} f(x) \, dx = \left[ \frac{5}{3} \cdot \frac{5}{3} x^2 \right]_{1}^{4} = \frac{5}{6} \cdot 4^2 - \frac{5}{6} \cdot 1^2 = 0
\]
The graph at the right is $f(x)$. The area bounded by the curve and the $x$–axis on the interval $[-3, 0]$ is 15.75 and the area bounded by the curve and the $x$–axis on the interval $[0, 2]$ is 5.3.

Using the information above, evaluate each integral.

A. $\int_{-3}^{0} f(x)\,dx = 15.75$
B. $\int_{-3}^{0} f(x)\,dx = -15.75$

C. $\int_{0}^{2} f(x)\,dx = -5.3$
D. $\int_{0}^{2} f(x)\,dx = 5.3$

E. $\int_{-3}^{0} f(x)\,dx = \frac{15.75 - 5.3}{10.45}$
F. $\int_{2}^{3} f(x)\,dx = -10.45$

G. If $\int_{-1}^{0} f(x)\,dx = 3$, then $\int_{-3}^{1} f(x)\,dx =$

$$\int_{-3}^{0} f(x)\,dx = \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx$$

$$15.75 = A + 3$$

A = 12.75

H. If $\int_{-1}^{0} f(x)\,dx = 3$ and $\int_{-2}^{-1} f(x)\,dx = 7.6$, then $\int_{-3}^{-2} f(x)\,dx =$

$$\int_{-3}^{0} f(x)\,dx = \int_{-3}^{-2} f(x)\,dx + \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx$$

$$15.75 = A + 7.6 + 3$$

A = 5.15

I. If $\int_{1}^{2} f(x)\,dx = -2.9$, then $\int_{0}^{1} f(x)\,dx =$

$$\int_{0}^{2} f(x)\,dx = \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

$$-5.3 = A + -2.9$$

A = -2.4

Let $g(x)$ be a new function such that $\int_{-2}^{3} g(x)\,dx = 19.4$ and $\int_{1}^{3} g(x)\,dx = 3.8$, find:

A. $\int_{-2}^{1} g(x)\,dx$

$$\int_{-2}^{3} g(x)\,dx = \int_{-2}^{1} g(x)\,dx + \int_{1}^{3} g(x)\,dx$$

$$19.4 = \int_{-2}^{1} + 3.8$$

$$\int_{-2}^{1} = 15.6$$

B. $\int_{-2}^{3} 5g(x)\,dx$

$$5 \int_{-2}^{3} g(x)\,dx = 5 \int_{-2}^{3} (19.4) = 97$$
1) Evaluate \( \int_1^4 3x + 2 \, dx \) by

A) Sketching the region bounded by the graph and the \( x \)-axis and evaluate using the area under the curve.

\[
\frac{1}{3}(5) + \frac{1}{2}(3)(9) \quad \frac{57}{2} = 28 \frac{1}{2}
\]

B) Using the First Fundamental Theorem of Calculus to evaluate the integral.

\[
\int_1^4 f(x) \, dx = F(4) - F(1)
\]

\[
\int_1^4 3x + 2 \, dx = \left[ \frac{3x^2}{2} + 2x \right]_1^4 = \left[ \frac{3 \cdot 4^2}{2} + 2(4) \right] - \left[ \frac{3 \cdot 1^2}{2} + 2(1) \right] = 24 + 8 - \frac{3}{2} - 2 = 28 \frac{1}{2}
\]

2) Evaluate \( \int_0^2 x^2 \, dx \) using the FTC.

\[
\frac{x^3}{3} \bigg|_0^2 = \frac{2^3}{3} - 0 = 8
\]

3) Evaluate \( \int_0^4 2x - 2 \, dx \) by

A) Sketching the region bounded by the graph and the \( x \)-axis and evaluate using the area under the curve.

\[-\frac{1}{2}(1)(2) + \frac{1}{2}(3)(6) = -1 + 9 = 8\]

B) Using the First Fundamental Theorem of Calculus to evaluate the integral.

\[
\int_0^4 2x - 2 \, dx = \left[ x^2 - 2x \right]_0^4 = \left[ 4^2 - 2(4) \right] - \left[ 0^2 - 2(0) \right] = 8
\]

4) Evaluate \( \int_{-2}^{2} x^2 \, dx \) = \( \frac{x^3}{3} \bigg|_{-2}^{2} = \frac{2^3}{3} - \left( -\frac{2}{3} \right) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} \)

Same as \( 2 \int_0^2 x^2 \, dx \)

Assignment #7: Page 308: 43–46, 55–62, Use the First Fundamental Theorem of Calculus to do problems 33–42
The First Fundamental Theorem of Calculus

If \( f(x) \) is continuous on \([a, b]\) and \( F(x) \) is the antiderivative of \( f(x) \), then

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a)
\]

Note: The result is a number!!!!!!

\[
\int_{-3}^{2} (6 - x - x^2) \, dx =
6x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \bigg|_{-3}^{2}
\]

\[
= \left[ 6 \cdot 2 - \frac{1}{2} \cdot 2^2 - \frac{1}{3} \cdot 2^3 \right] - \left[ 6(-3) - \frac{1}{2}(-3)^2 - \frac{1}{3}(-3)^3 \right]
\]

\[
= 12 - 4 - \frac{8}{3} - (-18 + \frac{9}{2})
\]

\[
= 12 - 4 - \frac{8}{3} + 18 - \frac{9}{2}
\]

\[
= 24 - \frac{37}{6}
\]

\[
= \frac{37}{6}
\]

\[
\int_{0}^{\pi/2} \sin x \, dx =
-\cos x \bigg|_{0}^{\pi/2}
\]

\[
= -\cos \frac{\pi}{2} - (-\cos 0)
\]

\[
= 0 + 1
\]

\[
\int_{0}^{\pi/2} \sec x \tan x \, dx =
\sec x \bigg|_{0}^{\pi/4}
\]

\[
\sec \frac{\pi}{4} - \sec 0
\]

\[
= \sqrt{2} - 1
\]

\[
\int_{0}^{\pi} 5 \sin x \, dx =
-5 \cos x \bigg|_{0}^{\pi/2}
\]

\[
= -5 \cos \frac{\pi}{2} - (-5 \cos 0)
\]

\[
= 0 + 5(1)
\]

\[
= 5
\]

Same as

\[
5 \int_{0}^{\frac{\pi}{2}} \sin x \, dx
\]
\[ \int_{1}^{2} \left( 3 - \frac{6}{x^2} \right) \, dx = \int_{1}^{2} 3 - 6x^{-2} \, dx \]

\[ 3x - 6x^{-1} \bigg|_{1}^{2} \]

\[ 3x + \frac{6}{x} \bigg|_{1}^{2} \]

\[ (3 \cdot 2 + \frac{6}{2}) - (3 \cdot 1 + \frac{6}{1}) \quad \text{AP Stop} \]

\[ \int_{0}^{1} \sqrt{x} \, dx = \int_{0}^{1} x^{\frac{1}{2}} \, dx \]

\[ \frac{2}{3} x^{\frac{3}{2}} \bigg|_{0}^{1} \]

\[ \frac{2}{3}, 1^{\frac{3}{2}} - \frac{2}{3}, 0^{\frac{3}{2}} \]

\[ \frac{2}{3} \]

\[ \int_{-4}^{4} |x^2 - 4| \, dx = \]

\[ \int_{-2}^{2} x^2 - 4 \, dx + \int_{2}^{4} -x^2 + 4 \, dx + \int_{4}^{-2} x^2 - 4 \, dx \]

\[ \frac{x^3}{3} - 4x \bigg|_{-2}^{-2} + \frac{-x^3}{3} + 4x \bigg|_{-2}^{2} + \frac{-x^3}{3} - 4x \bigg|_{2}^{4} \]

\[ \left[ \frac{(-2)^3}{3} - 4(-2) \right] - \left[ \frac{(-4)^3}{3} - 4(-4) \right] + \left[ -\frac{2^3}{3} + 4(2) \right] - \left[ -\frac{(-2)^3}{3} + 4(-2) \right] + \left[ -\frac{4^3}{3} - 4(3) \right] - \left[ -\frac{2^3}{3} - 4(2) \right] \]

\[ \text{STOP} \]
\[ \int_0^{2\pi} |\sin x| \, dx = \]
\[ = \int_0^\pi \sin x \, dx + \int_\pi^{2\pi} -\sin x \, dx \]
\[ = -\cos x \bigg|_0^\pi + \cos x \bigg|_\pi^{2\pi} \]
\[ = -(-1) + 1 - (-1) \]
\[ = 4 \]

If \( f(x) = 9x + \cos(x) \) and \( F \) is the antiderivative of \( f \), with \( F(0) = -4 \), then find \( F(3) \).

\[ \int_0^3 (9x + \cos x) \, dx = \left. \frac{9}{2} x^2 + \sin x \right|_0^3 \]
\[ F(3) - F(0) = \left. \frac{9}{2} x^2 + \sin x \right|_0^3 \]
\[ F(3) - (-4) = \left. \frac{9}{2} x^2 + \sin x \right|_0^3 \]
\[ F(3) - (-4) = \left. \frac{9}{2} \cdot 3^2 + \sin 3 \right|_0^3 - \left[ \left. \frac{9}{2} \cdot 0^2 + \sin 0 \right|_3 \right] \]
\[ F(3) = \frac{81}{2} + \sin 3 - (-4) \]

\( \int_1^9 f(x) \, dx = -5 \) and \( \int_1^9 f(x) \, dx = 6 \), then \( \int_1^4 f(x) \, dx = ? \) and \( \int_1^9 3f(x) - 8 \, dx = ? \)

\[ \int_1^9 f(x) \, dx = \int_1^4 f(x) \, dx + \int_4^9 f(x) \, dx \]
\[ -5 = \int_1^4 f(x) \, dx + 6 \]
\[ -11 = \int_1^4 f(x) \, dx \]

\[ \int_1^9 3f(x) - 8 \, dx = \int_1^9 3f(x) \, dx - \int_1^9 8 \, dx \]
\[ = 3 \left. \int_1^9 f(x) \, dx - 8x \right|_1^9 \]
\[ = 3(-5) - \left[ 8 \cdot 9 - 8 \cdot 1 \right] \]
\[ = -15 - 64 \]
\[ = -79 \]
1. Determine on what interval $y = e^x - x^3$ is increasing, for $-1 \leq x \leq 2$. Use your calculator.
Lesson 9

Topic: Leibniz Rule (2nd Fundamental Theorem of Calculus)\[ \frac{d}{dx} \left[ \int_a^x f(t) \, dt \right] = f(x) \]

Goal: Apply the Second Fundamental Theorem of Calculus to find the derivative of functions defined in terms of an integral.

The 2nd Fundamental Theorem of Calculus: If \( f \) is continuous on \([a, b] \) and \( u(x) \) and \( v(x) \) are differentiable functions of \( x \) whose value lie in \([a, b] \), then:

\[
\frac{d}{dx} \left[ \int_{u(x)}^{v(x)} f(t) \, dt \right] = f(v(x)) \frac{dv}{dx} - f(u(x)) \frac{du}{dx}
\]

Method I
Integrate then take the derivative.

\[
\int_{\pi/2}^x \cos t \, dt = \sin t \bigg|_{\pi/2}^x = \sin x^3 - \sin \frac{\pi}{2}
\]

\[
= \sin x^3 - 1 \quad \text{now take deriv}
\]

\[
\cos x^3 \cdot 3x^2 \quad \text{long way}
\]

Method II
Apply the 2nd Fundamental Theorem of Calculus.

\[
\frac{d}{dx} \int_{\pi/2}^x \cos t \, dt = \cos x^3 \cdot 3x^2 - \cos \frac{\pi}{2} \cdot 0 = 3x^2 \cos x^3
\]

Can't always integrate so Method 2 works better.

If \( f(x) = \int_{1/x^t}^x \frac{1}{x} \, dt \), then find \( f'(x) \).

\[
f'(x) = \frac{1}{x} \cdot 1 - \frac{1}{x} \cdot (-1x^{-2})
\]

\[
= \frac{1}{x} + x \cdot \frac{1}{x^2} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}
\]

If \( g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin t^2 \, dt \), then find \( g'(y) \).

\[
g'(y) = \sin(2\sqrt{y}) \cdot 2\sqrt{y} - \sin(\sqrt{y}) \cdot \frac{1}{2} \cdot \frac{1}{2}
\]

If \( h(x) = \int_{\cos x}^{1/x^2} \frac{1}{1-t^2} \, dt \), then find \( h'(x) \).

\[
h'(x) = \frac{\sin x}{1-(\sin x)^2} \cdot \cos x - \frac{1}{1-(\cos x)^2} \cdot -\sin x
\]

\[
= (t^2 + 1)^{\frac{1}{2}}
\]

If \( F(x) = \int_{3}^{2x} \sqrt{t^2 + 1} \, dt \), then find \( F'(x) \).

\[
F'(x) = \left[ (2x)^2 + 1 \right]^{\frac{1}{2}} \cdot 2 - \left[ 3^2 + 1 \right]^{\frac{1}{2}} \cdot 0
\]

\[
F'(x) = \left[ (2x)^2 + 1 \right]^{\frac{1}{2}} \cdot 2
\]
AP Calculus AB Y

Let \( f \) be the function that is continuous and differentiable on the interval \([0, 8]\) defined by \( f(x) = \int_0^x g(t) \, dt \). The graph of \( g \) is the piecewise function shown at the right.

A. Find \( f(1), f'(1), f''(1) \).

\[
f(1) = \int_0^1 g(t) \, dt = \frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}
\]

\[
f'(x) = g(x) \cdot 1 - g(x) \cdot 0 \quad f'(1) = g(1) = 3
\]

\[
f''(x) = g'(x) \quad \Rightarrow \quad f''(1) = g'(1) = 3
\]

The slope of \( g \) at \( x=1 \)

B. Find \( f(2), f'(2), f''(2) \).

\[
f(2) = \int_0^2 g(t) \, dt = \frac{1}{2} \cdot 2 \cdot 6 = 6
\]

\[
f'(x) = g(x) \quad f'(2) = g(2) = 6
\]

\[
f''(2) = g'(2) = \text{DNE \ b/c} \quad \lim_{x \to 2^-} g'(x) \neq \lim_{x \to 2^+} g'(x) \quad \text{sharp turn is a clue}
\]

C. Is \( x = 6 \) a maximum or minimum of \( f \)? Explain your reasoning.

\( x=6 \) is a max b/c \( f'(x) = g(x) \) and \( g \) goes from pos to neg at \( x=6 \) \( \Rightarrow \) \( f \) goes from inc to dec.

D. How many points of inflection does \( f \) have? Explain your reasoning.

\( g'(x) = f''(x) \quad \text{There is 1 poi \ b/c} \quad g \) goes from inc to dec at \( x=2 \) \( \Rightarrow \) \( g' \) goes from pos to neg.

C. What is the equation of the tangent line of \( f \) at \( x = 1 \)?

Point from A

\[
(1, \frac{3}{2}) \quad \text{Slope} \quad f'(1) = 3
\]

\[
\frac{y - \frac{3}{2}}{x - 1} = 3(x - 1)
\]

What is the value of \( x \) that maximizes the value of \( F(x) \) if \( F(x) = \int_x^{x+3} t(5-t) \, dt \)?

\[
F'(x) = (x+3)(5-(x+3)) - 1 - x(5-x) \cdot 1
\]

\[
F'(x) = (x+3)(-x+2) - 5x + x^2
\]

\[
F'(x) = -x^2 + 2x - 3x + 6 - 5x + x^2
\]

\[
0 = -6x + 6
\]

\[
x = 1
\]

Assignment #9: Page: 320: 21-24, 29-32 all, Handout
Assignment #10: Review Handout