Draw a slope field for \( \frac{dy}{dx} = x - 1 \)

\[ \frac{1}{0} \text{ is a hole} \]

\[ \frac{1}{0} \text{ is vert asymp} \]

Select a point, find the slope at that point, and graph the slope.

- At \((0,0)\)
  \[ \frac{dy}{dx} = 0 - 1 = -1 \]
  Slope at \((0,0)\)
- At \((0,1)\)
  \[ \frac{dy}{dx} = 0 - 1 = -1 \]
  All slopes on y-axis
- At \((1,0)\)
  \[ \frac{dy}{dx} = 1 - 1 = 0 \]
- At \((1,1)\)
  \[ \frac{dy}{dx} = 1 - 1 = 0 \]
  All at \(x = 1\)
- At \((2,0)\)
  \[ \frac{dy}{dx} = 2 - 1 = 1 \]
- At \((3,0)\)
  \[ \frac{dy}{dx} = 3 - 1 = 2 \]
- At \((2,1)\)
  \[ \frac{dy}{dx} = 2 - 1 = 1 \]
- At \((3,1)\)
  \[ \frac{dy}{dx} = 3 - 1 = 2 \]

Draw a slope field for \( \frac{dy}{dx} = x - y \).

- \((0,0)\)
  \[ 0 \]
- \((1,0)\)
  \[ 1 \]
- \((2,0)\)
  \[ 2 \]
- \((3,0)\)
  \[ 3 \]
- \((-1,0)\)
  \[ -1 \]
- \((-2,0)\)
  \[ -2 \]
- \((0,1)\)
  \[ -1 \]
- \((0,2)\)
  \[ -2 \]

Assignment 1: Handout
Topic: Slope Fields

Lesson 1

Goal: To be able to draw a slope field for a differential equation, identify a slope field for a differential equation, and sketch a solution curve given initial data.

Our goal is to draw what we call a slope field for a differential function. A slope field is a graph of the slopes of the tangents lines of a family of functions.

Before drawing a slope field, solve the differential equation.

\[
\frac{dy}{dx} = x - 1
\]

\[
y' = x - 1
\]

\[
dy = (x - 1) \, dx
\]

\[
\int dy = \int (x - 1) \, dx
\]

\[
y = \frac{x^2}{2} - x + C
\]

Sketch the graph(s) of the equation:

The graphs of the function for \( C = -5 \) to \( 5 \).

The Slope Fields for the same function.

Through \((1,3)\)
Topic: Separable Differential Equations

Goal: To solve a certain family of ordinary differential equations called "separable differential equations". Any equation with a derivative in it \( \frac{dy}{dx} \)

Given \( x^2 - y^2 = 5 \), find \( \frac{dy}{dx} \).

\[
2x - 2y \frac{dy}{dx} = 0 \quad \Rightarrow \quad 2x = 2y \frac{dy}{dx} \quad \Rightarrow \quad \frac{x}{y} = \frac{dy}{dx}
\]

An equation is **separable** if it can be written in the form \( f(y)dy = g(x)dx \) (Note: the x's are on one side and the y's are on the other side.)

**Example:**

\[
\frac{dy}{dx} = -\frac{4x}{9y}
\]

\[
9y \, dy = -4x \, dx
\]

\[
\int 9y \, dy = \int -4x \, dx
\]

\[
\frac{9y^2}{2} = -2x^2 + C
\]

**General Equation**

Let \( x = 1 \), \( y = 2 \)

\[
\frac{9 \cdot 2^2}{2} = -2(1)^2 + C
\]

\[
18 = -2 + C
\]

\[
C = 20
\]

\[
\frac{9y^2}{2} = -2x^2 + 20
\]

**Particular Solution**

\[
y = \pm \sqrt{\frac{-4x^2 + 40}{9}}
\]

Since \( y = 2 \), then take + \( \sqrt{\cdot} \)\n
\[
y = \sqrt{\frac{-4x^2 + 40}{9}}
\]

If \( y = -2 \), then take - \( \sqrt{\cdot} \)
Step 1: Get all of the \( y \)'s on one side and the \( x \)'s on the other side.

\[
\frac{dy}{dx} = x(x + 1) \text{ and } y(1) = 1
\]

\[
\int dy = \int x^2 + x \, dx
\]

\[
y = \frac{x^3}{3} + \frac{x^2}{2} + C
\]

\[
1 = \frac{1}{3} + \frac{1}{2} + C
\]

\[
6 = 2 + 3 + 6C
\]

\[
1 = 6C
\]

\[
C = \frac{1}{6}
\]

\[
y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{6}
\]

**Step 2:** Integrate both sides.

**Step 3:** Solve for \( C \) if you are given values for the variables.

**Step 4:** Solve for \( y \) (or just clean it up as is necessary)

\[
\frac{dy}{dx} = \frac{x}{y} \text{ and } y(0) = 1. \text{ Solve for } y.
\]

\[
y \, dy = x \, dx
\]

\[
\int y \, dy = \int x \, dx
\]

\[
\frac{y^2}{2} = \frac{x^2}{2} + C
\]

\[
\frac{1^2}{2} = \frac{0^2}{2} + C
\]

\[
C = \frac{1}{2}
\]

\[
\frac{dy}{dx} = \frac{y^2}{x}
\]

\[
\frac{1}{y^2} \, dy = \frac{1}{x} \, dx
\]

\[
\int \frac{1}{y^2} \, dy = \int \frac{1}{x} \, dx
\]

\[
-\frac{1}{y} = \ln |x| + C
\]

\[
-\frac{1}{y} = \ln |x| + C
\]

\[
\no \ x + y \text{ so solve for } y
\]
\[ \frac{dy}{dx} = xy \]

\[ \frac{1}{y} \, dy = x \, dx \]

\[ S \frac{1}{y} \, dy = S \, x \, dx \]

\[ \ln |y| = \frac{x^2}{2} + C \]

\[ e^{\ln |y|} = e^{\frac{x^2}{2} + C} \]

\[ y = e^{\frac{x^2}{2} + C} \]

\[ \text{or } y = e^{\frac{x^2}{2}} \cdot e^C \]

\[ \text{This is a constant} \]

\[ \frac{dy}{dx} = e^x \cdot y^3 \]

\[ \frac{1}{y^3} \, dy = e^x \, dx \]

\[ S \frac{1}{y^3} \, dy = S \, e^x \, dx \]

\[ \frac{y^{-2}}{-2} = e^x + C \]

\[ y^{-2} = -2e^x - 2C \]

\[ \frac{1}{y^2} = -2e^x - 2C \]

\[ y^2 = \frac{1}{-2e^x - 2C} \]

\[ y = \pm \sqrt{\frac{1}{-2e^x - 2C}} \]

\[ \frac{dy}{dx} = \frac{e^{2x}}{e^y} \text{ and } x = 0, y = 1 \]

\[ e^y \, dy = e^{2x} \, dx \]

\[ S e^y \, dy = S \, e^{2x} \, dx \]

\[ e^y = e^{\frac{2x}{2}} + C \]

\[ e^y = e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \]

\[ \ln e^y = \ln \left| e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \right| \]

\[ y = \ln \left| e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \right| \]

\[ e^y = e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \]

\[ \ln e^y = \ln \left| e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \right| \]

\[ y = \ln \left| e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \right| \]

\[ e = e^{\frac{2x}{2}} + e^{-\frac{1}{2}} \]

\[ c = e^{-\frac{1}{2}} \]

Assignment #2: Handout
Topic: Exponential Growth and Decay—Day 1

Lesson 3

Goal: Solve exponential growth and decay problems using the formula $A = A_0 e^{kt}$

Formula derived from Calculus definition: A quantity $y$ whose rate of change over time is proportional to the amount present.

\[
\frac{dy}{dt} = ky
\]

separate variables

\[
\frac{1}{y} \, dy = k \, dt
\]

$k$ is not a variable

\[
\int \frac{1}{y} \, dy = \int k \, dt
\]

$\ln |y| = kt + C$

$e^{\ln |y|} = e^{kt + C}$

$y = e^{kt + C}$

Abs value goes away b/c $e^{kt + C}$ will always be positive

$y = e^{kt} \cdot e^c$

exponent rules

$y = e^c \cdot e^{kt}$

$y = c \cdot e^{kt}$

constant

If $t = 0$

$y = c \cdot e^{k \cdot 0}$

$y = c$

$y = y_0 e^{kt}$ or

$A = A_0 e^{kt}$

$A_0$: Initial value

t: time

A: Ending value

k: Constant of proportionality

determines growth or decay

$k > 0$

growth \[
\]

$k < 0$

decay \[
\]
Example 1: The number of bacteria present when the experiment began was 1200. After 100 hours there were 2700 bacteria present. Assume exponential growth and find the time required for the number of bacteria to increase to 30,000.

\[
\frac{2700}{1200} = e^{100k} \\
\ln \left( \frac{9}{4} \right) = 100k \\
\frac{\ln \left( \frac{9}{4} \right)}{100} = k \\
\frac{2700}{1200} = e^{100k} \\
\frac{30,000}{1200} = e^{100k} \\
\ln \left( \frac{9}{4} \right) = k \\
\frac{\ln 25}{\ln \left( \frac{9}{4} \right)} = t \\
\ln 25 = \ln e^{100k} \\
\ln 25 = 100k \\
\ln 25 = kt \\
\ln 25 = t \\
t \approx 396.936 \text{ hrs}
\]

Example 2: In a certain research experiment, a population of fruit flies increases according to the law of exponential growth. If there were 180 flies after the second day of the experiment and 300 flies after the fourth day, how many flies were in the original population?

\[
180 = A_0 e^{k \cdot 2} \\
300 = A_0 e^{k \cdot 4} \\
\frac{180}{e^{2k}} = A_0 \\
\frac{300}{e^{4k}} = A_0 \\
\frac{180}{e^{2k}} = \frac{300}{e^{4k}} \\
e^{2k} = \frac{5}{3} \\
\ln e^{2k} = \ln \left( \frac{5}{3} \right) \\
2k = \ln \left( \frac{5}{3} \right) \\
k = \frac{\ln \left( \frac{5}{3} \right)}{2}
\]
Example 3: Let $y$ represent the mass of a particular radioactive element whose HALF-Life is 25 years. (In other words, if we began with 1 gram of the element, only 1/2 gram would remain after 25 years, 1/4 gram after 50 years, etc.) How much of a 1 gram mass would remain after 15 years?

$$\frac{1}{2} = 1e^{k \cdot 25}$$

$$\ln \frac{1}{2} = 25k$$

$$\ln \frac{1}{2} = \frac{25k}{25} = k$$

$$A = 1e^{k \cdot 15}$$

$$A = e^{\frac{\ln \frac{1}{2}}{25} \cdot 15}$$

$$A \approx 0.660 \text{ g}$$

Assignment #3: Handout Day 1
Example 4: If the half-life of disappearium is 15.8 minutes, what percent of the amount present now will be remaining after 1 hour?

\[ A_0 = 1 \quad t = 15.8 \]

\[ A = \frac{1}{2} \quad t = 60 \]

\[ \ln \frac{1}{2} = 15.8K \]

\[ \ln \frac{1}{2} = \ln e^{15.8K} \]

\[ \frac{\ln \frac{1}{2}}{15.8} = K \]

\[ A = 1 \cdot e^{60K} \]

\[ A = 0.07197 \quad A = 7.197\% \]
Goal: Find the position, velocity, and acceleration functions from information given and deduce facts using the three functions.

\[ x(t) \]
\[ s(t) = \text{position equation} \]
\[ v(t) = \text{velocity equation} = s'(t) \text{ ft/sec} \]
(how fast position changes)
\[ \int v(t) \, dt = s(t) + C \]
\[ a(t) = \text{acceleration equation} = s''(t) = v'(t) = \text{ft/sec}^2 \]
(how fast velocity changes)
\[ \int a(t) \, dt = v(t) + C \]

speed = |velocity|

\[
\text{average velocity} = \frac{s(b) - s(a)}{b - a}
\]
\[
\text{average acceleration} = \frac{v(b) - v(a)}{b - a}
\]

Notes:
\[
\int_a^b a(t) \, dt = \frac{v(b) - v(a)}{b - a}
\]

\[ v(t) = 0 \quad \text{STOPPED!} \]
(Position is not changing at that instant)

\[ v(t) < 0 \quad \text{Moves in Negative direction (down or left)} \]
\[ v(t) > 0 \quad \text{Moves in Positive direction (up or right)} \]
\[ a(t) = 0 \quad \text{Constant velocity} \]

Speeding Up
\[ a(t) > 0 \quad \text{and } v(t) > 0 \quad \text{or } a(t) < 0 \quad \text{and } v(t) < 0 \]

Slowing Down
\[ a(t) > 0 \quad \text{and } v(t) < 0 \quad \text{or } a(t) < 0 \quad \text{and } v(t) > 0 \]

Let \[ s(t) = t^3 - 6t^2 \] be the position measured in feet of a particle.

Find velocity, speed, and acceleration at \( t = 1, 2, \) and \( 4 \) sec. of the particle.

\[ v(t) = 3t^2 - 12t \]
\[ v(1) = 3(1)^2 - 12(1) = -9 \text{ ft/sec} \]
(speed = 9 ft/sec)
\[ v(2) = 3(2)^2 - 12(2) = -12 \text{ ft/sec} \]
(speed = 12 ft/sec)
\[ v(4) = 3(4)^2 - 12(4) = 0 \text{ ft/sec} \]
(speed = 0 ft/sec)

\[ a(t) = 6t - 12 \]
\[ a(1) = 6(1) - 12 = -6 \text{ ft/sec}^2 \]
\[ a(2) = 6(2) - 12 = 0 \text{ ft/sec}^2 \]
\[ a(4) = 6(4) - 12 = 12 \text{ ft/sec}^2 \]

\[ s(t) = t^3 - 6t^2 \]

<table>
<thead>
<tr>
<th>velocity</th>
<th>( v(t) = 3t^2 - 12t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 3(t - 4) )</td>
</tr>
<tr>
<td>( t = 0 )</td>
<td>( t = 4 )</td>
</tr>
</tbody>
</table>

\[ s(t) \]

<table>
<thead>
<tr>
<th>Left dec</th>
<th>Right inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( \text{neg} )</td>
<td>( \text{pos} )</td>
</tr>
</tbody>
</table>

2019-2020
Let \( v(t) = 6t^2 - 42t + 60 \) be the velocity of a particle in feet per second.

A) Find the intervals on which the object (particle) moves left and right.

\[
\begin{align*}
0 &= 6t^2 - 42t + 60 \\
0 &= t^2 - 7t + 10 \\
0 &= (t - 5)(t - 2) \\
t &= 2, 5
\end{align*}
\]

\( v(t) < 0 \) \quad \text{The particle is moving right on } (0, 2) \text{ } \quad \text{b/c } v(t) > 0
\]

\( v(t) > 0 \) \quad \text{and left on } (2, 5) \text{ } \quad \text{b/c } v(t) < 0.

B) If \( s(t) \) is the position function of the particle and \( s(1) = 44 \), find the position function.

\[
\begin{align*}
\int v(t) \, dt &= s(t) \\
\int (6t^2 - 42t + 60) \, dt &= s(t) \\
s(t) &= 2t^3 - 21t^2 + 60t + C
\end{align*}
\]

\( s(1) = 2(1)^3 - 21(1)^2 + 60(1) + C = 44 \), so \( C = 3 \).

C) What is the average velocity of the particle on the interval \([1, 4]\)?

\[
\frac{s(4) - s(1)}{4 - 1} = \frac{2(4)^3 - 21(4)^2 + 60(4) + 3 - (2(1)^3 - 21(1)^2 + 60(1) + 3)}{3} = -3 \text{ ft/sec}
\]

D) When will the instantaneous velocity equal the average velocity on the interval \([1, 4]\)? \text{Calculator.}

\[
v(t) = 6t^2 - 42t + 60 = -3
\]

\( t = 2.177 \text{ sec} \)

E) What is the average acceleration of the particle on the interval \([1, 4]\)?

\[
\frac{v(4) - v(1)}{4 - 1} = \frac{12t - 42 - 24}{3} = -12 + \frac{t}{3} \text{ ft/sec}^2
\]

F) Find \( a(t) \) the acceleration function of the particle.

\( v'(t) = a(t) = 12t - 42 \)

G) When will the average acceleration equal the instantaneous acceleration on the interval \([1, 4]\)?

\[
-12 = a(t) = 12t - 42 \\
30 = 12t \\
t = \frac{30}{12} = \frac{5}{2} \text{ sec}
\]

H) On what intervals is the particle speeding up or slowing down?

\[
\begin{align*}
a(t) &= 0 \\
0 &= 12t - 42 \\
42 &= 12t \\
t &= \frac{42}{12} = \frac{7}{2}
\end{align*}
\]

The particle is slowing down on \((0, 2) \cup \left(\frac{7}{2}, 5\right)\) \text{ b/c } v(t) + a(t) have different signs

The particle is speeding up on \((2, \frac{7}{2}) \cup (5, \infty)\) \text{ b/c } v(t) + a(t) have the same sign.
Let \( a(t) = 6t - 12 \) be the acceleration of a particle in ft per second\(^2\).

A) If \( v(t) \) is the velocity function of the particle and \( v(1) = 3 \), find the velocity function.

\[
v(t) = \int a(t) \, dt \quad v(1) = 3 \cdot 1^2 - 12 \cdot 1 + C \\
v(t) = 3t^2 - 12t + C \quad 3 = 3 - 12 + C \quad v(t) = 3t^2 - 12t + 12 \\
12 = C
\]

B) Find the intervals on which the object (particle) moves left and right.

\[
0 = 3t^2 - 12t + 12 \quad t = 1, 3 \\
0 = t^2 - 4t + \quad v(t) + - + \quad \text{The particle is moving left on} \\
0 = (t - 2)(t - 2) \quad \text{right on} \quad (0, 2) \quad v(2, \infty) \quad \text{b/c} \quad v(t) > 0
\]

C) What is the average acceleration of the particle on the interval \([0, 5]\)?

\[
\frac{v(5) - v(0)}{5 - 0} \quad \frac{v(5) = 75 - 60 + 12}{v(0) = 12} \quad \frac{27 - 12}{5} \quad \frac{3}{5} \text{ ft/sec}^2
\]

D) When will the average acceleration equal the instantaneous acceleration on the interval \([0, 5]\)?

\[
\frac{3}{a(t)} \quad 3 = 6t - 12 \quad 15 = 6t \quad t = \frac{15}{6} = \frac{5}{2} \text{ sec}
\]

E) If \( x(t) \) is the position function of the particle and \( x(1) = 5 \), find the position function.

\[
x(t) = \int v(t) \, dt \quad 5 = 1^3 - 6 \cdot 1^2 + 12 \cdot 1 + C \\
x(t) = t^3 - 6t^2 + 12t + C \quad 5 = 7 + C \quad C = -2 \quad x(t) = t^3 - 6t^2 + 12t - 2
\]

F) What is the average velocity of the particle on the interval \([0, 5]\)?

\[
\frac{x(5) - x(0)}{5 - 0} \quad x(5) = 125 - 150 + 60 - 2 = 33 \quad \frac{33 - (-2)}{5} \quad 7 \text{ ft/sec}
\]

G) When will the instantaneous velocity equal the average velocity on the interval \([0, 5]\)? \textbf{Calculator.}

\[
v(t) \\
3t^2 - 12t + 12 = 7 \quad 3t^2 - 12t + 5 = 0 \quad t = 0.472 \text{ sec} \quad t = 3.528 \text{ sec}
\]

H) On what intervals is the particle speeding up or slowing down?

\[
a(t) = 0 \quad \text{The particle is slowing down on} \\
6t - 12 = 0 \quad 6t = 12 \quad t = 2 \quad \text{b/c} \quad v(t) + a(t) \quad \text{have different signs.} \\
a(t) + \quad \text{The particle is speeding up on} \\
0 \quad (0, 2) \quad \text{b/c} \quad v(t) + a(t) \quad \text{have same signs.}
\]
A ball is thrown upward from a 160 ft high cliff with an initial velocity of 48 ft/sec. Use the equation \( h(t) = -16t^2 + v_0t + h_0 \).

\[ x(t) = \]

A) find position equation

\[ x(t) = -16t^2 + 48t + 160 \]

B) find max height

\[ v(t) = -32t + 48 \]
\[ 0 = -32t + 48 \]
\[ 32t = 48 \]
\[ t = \frac{48}{32} = \frac{3}{2} \text{ sec} \]

\[ x\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 160 \]
\[ = -16\cdot \frac{9}{4} + 72 + 160 \]
\[ = -36 + 72 + 160 \]
\[ = 196 \text{ ft} \]

vertex of parabola

\[ x = -\frac{b}{2a} \Rightarrow t = -\frac{b}{2a} = -\frac{-48}{2\cdot(-16)} = \frac{3}{2} \]

C) find when the ball hits the ground

\[ 0 = -16t^2 + 48t + 160 \]
\[ 0 = t^2 - 3t - 10 \]
\[ 0 = (t-5)(t+2) \]
\[ t = 5 \quad t = -2 \text{ sec} \]

D) find velocity of the ball at the time it hits the ground

\[ v(5) = -32(5) + 48 \]
\[ = -160 + 48 \]
\[ = -112 \text{ ft/sec} \]

speed would be 112 ft/sec
The piecewise function at the right represents the velocity of a particle for the interval [0, 5] in meters per year.

Determine on what intervals the particle is moving to the left and right and justify your conclusions.

The particle is moving to the right on (0, 3) b/c \( v(t) > 0 \)
and left on (3, 5) b/c \( v(t) < 0 \)

Find \( a(1) \) and \( a(3) \).

\[
a(1) = v'(1) = 1 \text{ m/yr}^2
\]
\[
a(3) = v'(3) = -4 \text{ m/yr}^2
\]

Determine if the speed of the particle is increasing or decreasing at \( t = 1, t = 2.5, \) and \( t = 4 \).

\( v(1) > 0 \); \( a(1) < 0 \), so the speed of the particle is inc
\( v(2.5) > 0 \) \( a(2.5) > 0 \) so the speed of the particle is dec
\( v(4) < 0 \), \( a(4) < 0 \) so the speed of the particle is inc

Explain why \( a(2) \) does not exist.

\[
\lim_{t \to 2^-} a(t) \neq \lim_{t \to 2^+} a(t)
\]

Assignment #5: Handout
**Topic: Total Distance**

**Lesson 6**

**Goal:** Find the total distance traveled by a particle along a linear path.

One way to find the total distance of a particle that is traveling along a linear path is to use the velocity function.

**Step 1:** Find \( v(t) \) -- tells us when the particle has stopped and the direction the particle is moving.

**Step 2:** Find the position at each moment in time that it changed direction.

**Step 3:** Make a diagram to see the total distance.

Find the total distance traveled by a particle if the position of the particle is given by the function \( s(t) \) over the time period specified.

If \( s(t) = 2t^3 - 9t^2 + 12t \) for \( t = 0 \) to \( t = 4 \).

\[
\begin{align*}
V(t) &= 6t^2 - 18t + 12 \\
0 &= 6t^2 - 18t + 12 \\
0 &= t^2 - 3t + 2 \\
0 &= (t - 2)(t - 1) \\
t &= 1, 2 \\
&= 1 \quad _t \quad 2 \quad \quad 1 \quad 2 \quad 4 \\
&= _{\text{sec}} \quad 0 \quad 1 \quad 2 \quad 4 \\
&= _{\text{Particle Stopped}} \\
\end{align*}
\]

**Position**

\[
\begin{align*}
S(0) &= 0 \\
S(1) &= 2 - 9 + 12 = 5 \text{ units right} > v(t) > 0 \\
S(2) &= 2 \cdot 2^3 - 9 \cdot 2^2 + 12 \cdot 2 = 4 \text{ un left} \quad > v(t) < 0 \\
S(4) &= 2 \cdot 4^3 - 9 \cdot 4^2 + 12 \cdot 4 = 32 \text{ un right} \quad > v(t) > 0 \\
\end{align*}
\]

\[
\begin{align*}
S(0) &= 0 \quad > 5 \text{ un} \\
S(1) &= 5 \quad > 5 \text{ un} \\
S(2) &= 4 \quad > 1 \text{ un} \\
S(4) &= 32 \quad > 28 \text{ un} \\
34 \text{ Total units} \quad \text{Traveled} \\
\end{align*}
\]

**Position**

\[
\begin{align*}
S(0) &= 0 \quad > 4 \text{ units} \\
S(2) &= 8 - 12 = -4 \quad > 20 \text{ units} \\
S(4) &= 64 - 48 = 16 \quad 24 \text{ units travelled} \\
\end{align*}
\]
If \( v(t) = 6t - 6 \) is the velocity function of a particle, then find:

A) If \( x(t) \) is the position function of the particle and \( x(1) = 6 \), then find \( x(t) \).

\[
\int v(t) \, dt = x(t) + C \quad x(t) = 3t^2 - 6t + C
\]

\[ 6 = 3 \cdot 1^2 - 6 \cdot 1 + C \]

\[ 9 = C \quad x(t) = 3t^2 - 6t + 9 \]

B) The total distance traveled by the particle for \( t = 0 \) to \( t = 3 \).

\[
\begin{align*}
0 &= 6t - 6 & v(t) &= - \quad + \\
\text{at} &= 1 & \text{Position} & x(0) = 9 \quad > 3 \text{ un} \\
\text{at} &= 1 & x(1) = 6 \quad > 12 \text{ un} \\
\text{at} &= 3 & x(3) = 18 \quad 15 \text{ un travelled}
\end{align*}
\]

C) Determine on what intervals for \( t \geq 0 \) when the particle is speeding up. Justify your solution.

\[ a(t) = 6 \] so \( a(t) \) is always pos. 

The particle is speeding up on \((1, 3)\) because \( v(t) > 0 \) and \( a(t) > 0 \).
We need to be able to interpret the total distance traveled from a graph of the velocity function.

The velocity function tells you when the particle is moving left or right. The area bounded by the graph and the x-axis of the velocity function represents the distance the particle traveled.

Draw the graph of $v(t) = 6t^2 - 18t + 12$ for $t = 0$ to $t = 4$.

Use the graph to determine at what time interval the particle is moving to the left.

The particle moves right on $(0, 1)$ and $(2, 4)$ b/c $v(t) > 0$

The particle moves left on $(1, 2)$ b/c $v(t) < 0$.

How can we find the total distance the particle traveled?

$$\int_0^1 v(t) \, dt - \int_1^2 v(t) \, dt + \int_2^4 v(t) \, dt$$

This will be negative so negate the negative

$$\left[ 2t^3 - 9t^2 + 12t \right]_0^1 - \left[ 2t^3 - 9t^2 + 12t \right]_1^2 + \left[ 2t^3 - 9t^2 + 12t \right]_2^4$$

$$2 - 9 + 12 - 0 - 16 + 36 - 2 + 9 + 12 = 34$$

Video is wrong last part of ex

Calculator - Math

$$\int_0^4 |v(t)| \, dt = 34$$

Assignment 6: Handout