Goal: To approximate the value of a function using the tangent and secant line.

Find the equation of the tangent line to \( f(x) = \sqrt{x} e^{x-1} \) at \( x = 1 \).

This is called the **Linearization** of the function.

**Point** \( f(1) = \sqrt{1} e^{1-1} = 1 \)

**Slope**

\[
\begin{align*}
f'(x) &= \frac{1}{2} x^{-\frac{1}{2}} e^{x-1} + x^\frac{1}{2} e^{x-1} \\
f'(1) &= \frac{1}{2} (1)^{-\frac{1}{2}} e^{1-1} + 1^\frac{1}{2} e^{1-1} \\
&= \frac{1}{2} + 1 = \frac{3}{2}
\end{align*}
\]

\( y - 1 = \frac{3}{2} (x - 1) \)

or

\( y = \frac{3}{2} (x - 1) + 1 \)

Use the tangent line to approximate \( f(1.1) \).

\[
\begin{align*}
y = \frac{3}{2} (1.1 - 1) + 1 &= \frac{3}{20} + 1 = \frac{23}{20} = 1.15
\end{align*}
\]

Is the approximation less than or greater than the actual value? Explain.

**The approximation is less b/c the tangent line is below \( f(x) \). This is because \( f(x) \) is concave up. (smiling)**

How far off from the actual value is your approximation?

**Actual** \( f(1) = \sqrt{1} e^{1-1} = 1 \)

**Approx** \( f(1) = 1.159113038 \)

**Error** \( |\text{Actual} - \text{Approx}| = |1 - 1.159113038| = 0.0091130376 \)

Use the tangent line to approximate \( f(2) \). How far off from the actual value is the approximation? Explain why the tangent line is not a good method to approximate \( f(2) \).

\[
\begin{align*}
y = \frac{3}{2} (2 - 1) + 1 &= \frac{5}{2} \approx \text{approx} \\
f(x) &= \sqrt{2} e^{2-1} = 3.844231028 \text{ (Actual)}
\end{align*}
\]

**Error** \( 1.344231028 \) see above graph

**Approx is only good if you are close to the point of tangency.**

Draw the secant line through \( x = 1 \) and \( x = 2 \). If you found the equation of the secant line and used that to approximate the value of the function at \( x = 1.1 \), would the approximation be less than or greater than the actual value? Explain.

greater b/c the secant line is above \( f(x) \)

Because \( f(x) \) is concave up, the secant line is above.

Assignment #1 p. 213 # 9-13, 45, 49, 53, 57

use linearization to estimate \( f(\Delta x) \)

compute error

2018-2019
Tangent Line Approximation

Equation of a Line: \[ y - y_1 = m(x - x_1) \]

Equation of the tangent line of \( f(x) \).

\[ y - f(a) = f'(a)(x - a) \] at the point \((a, f(a))\)

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta y \)?

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta x \)?

\[ \Delta f = f'(a) \Delta x \quad \text{or} \quad dy = f'(a) \, dx \]

We can rewrite the tangent line equation to look like

\[ \frac{\Delta y}{\Delta x} = f'(a) \]

If \( dx \) is small, then the linear approximation using the tangent line is approximately equal to the change in the function \( f \), also known as \( \Delta y \) or \( \Delta f \).

Given \( (x) = \frac{1}{x}, a = 10, \) and \( \Delta x = 0.2 \). Find \( \Delta f \). Then compute the error and percent error.

\[ \Delta f = f'(a) \Delta x \]

\[ f(x) = x^{-1} \]

\[ f'(x) = -1x^{-2} \]

\[ f'(10) = -\frac{1}{10^2} \]

\[ = -\frac{1}{100} \]

\[ \Delta f = -\frac{1}{100}(0.2) \]

\[ \Delta f = -0.002 \]

**Approx Change**

\[ Actual \ Change \ in \ f \]

\[ f(10.2) - f(10) \]

\[ = \frac{1}{10.2} - \frac{1}{10} \]

\[ = -0.0019607843 \]

**Error = |Actual - Approx|**

\[ = 0.0000392156 \]

**Percent Error = \left| \frac{\text{error}}{\text{Actual}} \right| \times 100**

\[ = 2\% \]

The approx is less than the actual value b/c \( f \) is concave up.
Tangent Line Approximation

Equation of a Line: \[ y - y_1 = m(x - x_1) \]

Equation of the tangent line of \( f(x) \):
\[ y - f(a) = f'(a)(x - a) \] at the point \((a, f(a))\)

What is \( \frac{\Delta y}{\Delta x} \)?

so \( \frac{\Delta y}{\Delta x} = \frac{\text{change in } f}{\text{change in } x} \)

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta y \)?

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta x \)?

\[ \Delta f = f'(a) \cdot \Delta x \quad \text{or} \quad dy = f'(a) \cdot dx \]

If \( dx \) is small, then the linear approximation using the tangent line is approximately equal to the change in the function \( f \) (\( \Delta y \) or \( \Delta f \)), so

Given \( x = \frac{1}{x}, a = 10, \) and \( \Delta x = 0.2 \). Find \( \Delta f \). Then compute the error and percent error.

\[ \Delta f = f'(a) \Delta x \]

\[ f(x) = x^{-1} \]
\[ f'(x) = -1 \cdot x^{-2} \]
\[ f'(10) = -1 \cdot 10^{-2} \]
\[ f'(10.2) \]

The change in \( f \) from \( x = 10 \) to \( x = 10.2 \)

Given \( f(x) = \sqrt{1 + x}, a = 4, \) and \( \Delta x = 0.3 \). Find \( \Delta f \). Then find the error and the percentage error.

\[ f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} \]
\[ f'(4) = \frac{1}{2}(1+4)^{-\frac{1}{2}} \]
\[ f'(4.3) \]

Assignment #1: Linear Approximation—Linearization

Page 213: Preliminary Question #3 (very top of page, NOT in Exercises). Exercise #25, 45, 54
### Tangent Line Approximation

**Equation of a Line:**
\[ y - y_1 = m(x - x_1) \]

**Equation of the tangent line of** \( f(x) \):
\[ y - f(a) = f'(a)(x - a) \] at the point \((a, f(a))\).

What is \( \frac{\Delta y}{\Delta x} \)?
\[ \frac{\text{change in } y}{\text{change in } x} \]
so, \[ \frac{\Delta y}{\Delta x} = f'(a) = \frac{dy}{dx} \]

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta y? \)
\[ y - f(a) \]

In the equation \( y - f(a) = f'(a)(x - a) \) what is \( \Delta x? \)
\[ x - a \]

We can rewrite the tangent line equation to look like
\[ \Delta y = f'(a) \Delta x \quad \text{or} \quad dy = f'(a) \, dx \]

If \( dx \) is small, then the linear approximation using the tangent line is approximately equal to the change in the function \( f(\Delta y \text{ or } \Delta f) \), so
\[ \Delta f \approx f'(a) \Delta x \]

**The change in the vertical distance between 2 points** \( f(x) \) and \( f(x + \Delta x) \)

Given \( f(x) = \frac{1}{x} \), \( a = 10 \), and \( \Delta x = 0.2 \). Find \( \Delta f \). Then compute the error and percent error.

\[
f(x) = x^{-1} \quad f'(x) = -1x^{-2} \quad f'(10) = -1(10)^{-2} = \frac{-1}{100} \]

**Slope of the Tangent Line at** \( x = 10 \)

\[ \Delta f = \frac{-1}{100} (0.2) = -0.002 \]

**Actual Change**
\[ f(10.2) - f(10) = -0.0019607843 \]

**Approx Change**
\[ \Delta f = -0.002 \]

**Error**
\[ \text{error} = 0.00003923568628 \]

**Percent Error**
\[ \frac{\text{error}}{\text{actual}} \times 100\% = 2\% \]

Given \( f(x) = \sqrt{1 + x} \), \( a = 4 \), and \( \Delta x = 0.3 \). Find \( \Delta f \). Then find the error and the percentage error.

\[
f'(x) = \frac{1}{2} (1 + x)^{-\frac{1}{2}} \quad f'(4) = \frac{1}{2} (1 + 4)^{-\frac{1}{2}} = \frac{1}{2\sqrt{5}} \]

\[ \Delta f = \frac{1}{2\sqrt{5}} (0.3) \]

**Actual**
\[ f(4.3) - f(4) = 0.0661049097 \]

**Approx**
\[ \Delta f = \frac{1}{2\sqrt{5}} (0.3) \]

**Error**
\[ 0.0009771296 \]

**Percent Error**
\[ 1.478\% \]

---

**Assignment #1: Page: 213**

Use the directions for 9–12 for these: 9, 10, 12, 13, 15, also do 45, 53, 57

*Use linearization to estimate* \( f(a + \Delta x) \) 9–12, 45, 49, 53, 57

*Compute error* 2018-2019
1. \(5x^4 - 7x^2 = 0\)
   \[x^2(5x^2 - 7) = 0\]
   \[x = \pm \sqrt{\frac{7}{5}}\]

2. \(\sin x - 2 \sin x \cos x = 0\), for \(0 \leq x \leq 2\pi\).
   \[\sin x (1 - 2 \cos x) = 0\]
   \[\sin x = 0\]
   \[x = 0, \pi, 2\pi\]

   \[1 - 2 \cos x = 0\]
   \[\cos x = \frac{1}{2}\]
   \[x = \frac{\pi}{3}, \frac{2\pi}{3}\]

3. \(x^{\frac{1}{3}} + 1 = 0\)
   \[x^{\frac{1}{3}} = -1\]
   \[(x^{\frac{1}{3}})^3 = (-1)^3\]
   \[x = -1\]
Goal: To describe the behavior of a function on an interval.

Extrema (extremum): extreme values

Minimum and Maximum are extrema.

The Extreme Value Theorem (EVT): If $f$ is continuous on a closed interval $[a, b]$, then $f$ has both a minimum and maximum on the interval. (existence theorem)

$y = x^2 - 3, [-2, 1]$  $y = x^3 - 3x^2$

Finding Extrema on a Closed Interval—this is our task!!!

Extrema on a closed interval occur at the endpoints or at critical numbers.

Critical Numbers:

$f'(c) = 0$ OR $f$ is not differentiable at $c$:

(sharp turn or cusp) $\lim_{x \to c^-} f'(x) \neq \lim_{x \to c^+} f'(x)$

Derivative undefined (vertical tangent lines)

then $c$ is a critical number.

If $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$, find the absolute maximum and absolute minimum.

Critical # of $f'$ is zero.

Find $f' = 0$  

Check Endpoints

$f'(x) = 12x^3 - 12x^2$

$f(-1) = 3(-1)^4 - 4(-1)^3 = 7$

$f(2) = 3(2)^4 - 4(2)^3 = 48 - 32 = 16$

$x = 0$  $x = 1$

$f(0) = 0$

$f(1) = 3 - 4 = -1$

The abs max is 16 when $x = 2$

The abs min is -1 when $x = 1$
Find the absolute maximum and minimum on the closed intervals.

\( f(x) = 2x - 3x^{\frac{2}{3}} \) on the interval \([-1, 3]\)

- Critical Points
  - \( f'(x) = 0 \)
  - \( f'(x) \) is undefined at \( x = 0 \)
  - \( \frac{2}{x^{\frac{2}{3}}} = 2 \)
  - \( 2 = 2x^{\frac{1}{3}} \)
  - \( 1 = x^{\frac{1}{3}} \)
  - \( x = 1 \)

\[ f(1) = 2(1) - 3(1)^{\frac{2}{3}} = -1 \]

\( f(-1) = 2(-1) - 3(-1)^{\frac{2}{3}} = -2 - 3 = -5 \) Abs Min

Endpoints
- \( x = -1, 3 \)
- \( x = 0, 1 \)

\( f(0) = 0 \) Abs Max

\( f(3) = 2(3) - 3(3)^{\frac{2}{3}} = -0.24 \)

Next...

\( f(x) = 2 \sin x - \cos 2x \) \([0, 2\pi]\)

- Critical Points
  - \( f'(x) = 2\cos x - (\sin 2x) 2 \)
  - \( = 2\cos x + 2\sin 2x \)
  - \( 0 = 2\cos x + 2\sin 2x \)
  - \( 0 = 2\cos x + (2\sin x \cos x) \)
  - \( 0 = 2\cos x (1 + 2\sin x) \)

- \( 2\cos x = 0 \)
- \( 1 + 2\sin x = 0 \)
- \( \cos x = 0 \)
- \( \sin x = -\frac{1}{2} \)
- \( x = \frac{\pi}{2}, \frac{3\pi}{2} \)
- \( x = \frac{7\pi}{6}, \frac{11\pi}{6} \)

\( f\left(\frac{\pi}{2}\right) = 2\sin \frac{\pi}{2} - \cos (2 \cdot \frac{\pi}{2}) = 2 - (-1) = 3 \) Abs Max

\( f\left(\frac{3\pi}{2}\right) = 2\sin \frac{3\pi}{2} - \cos (2 \cdot \frac{3\pi}{2}) = -2 - (-1) = -1 \)

\( f\left(\frac{7\pi}{6}\right) = 2\sin \frac{7\pi}{6} - \cos (2 \cdot \frac{7\pi}{6}) = -1 - \left(\frac{1}{2}\right) = -\frac{3}{2} \) Abs Min

\( f\left(\frac{11\pi}{6}\right) = 2\sin \frac{11\pi}{6} - \cos (2 \cdot \frac{11\pi}{6}) = -1 - \left(\frac{1}{2}\right) = -\frac{3}{2} \)

Assignment #2: Page: 222: 1, 3, 7, 16, 18, 29, 36, 39, 46, 50
Given: \( x(t) = t^3 - 4t + 9 \) is the position function where \( t \) is time in seconds and \( x(t) \) is measured in feet.

Find the average velocity on the interval [2, 4].

\[
\frac{x(4) - x(2)}{4 - 2} = \frac{4^3 - 4(4) + 9}{2}
\]

\[
x(4) = 4^3 - 4(4) + 9 = 64 - 16 + 9 = 57
\]

\[
x(2) = 2^3 - 4(2) + 9 = 8 - 8 + 9 = 9
\]

\[
\text{Average Velocity} = \frac{57 - 9}{2} = 24 \text{ ft/sec}
\]

What is the absolute maximum and minimum value of \( f(x) = x^3 - 3x + 9 \) on the interval \([0, 3]\)? Justify.

\[
f'(x) = 3x^2 - 3
\]

\[
0 = 3x^2 - 3
\]

\[
x = 1
\]

\[
x = -1 \text{ not in interval}
\]

\[
f(0) = 9
\]

\[
f(3) = 3^3 - 3(3) + 9 = 27 \text{ Abs Max}
\]

The justify is the math that shows max and min.
The Mean Value Theorem, MVT, is the general case of Rolle’s Theorem.

**MVT**

Let \( f \) be continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a, b)\), then there exists a number \( c \) in \((a, b)\) such that

\[
 f'(c) = \frac{f(b) - f(a)}{b - a}
\]

The graph of \( f(x) = -x^2 + 4x \) on the interval \([0, 3]\) is shown in the graph at the right.

1. Using the interval \([0, 3]\), \( a = 0 \) and \( b = 3 \).

2. What are the coordinates of the points for the given values of \( a \) and \( b \).

\[
(0, 0) \quad (3, 3)
\]

3. Draw the secant line through the two points and find the slope of the line.

\[
\text{slope} = \frac{3-0}{3-0} = 1
\]

4. Now, draw a line tangent to the graph that is also parallel to the secant line. From the graph, approximate the coordinates of the point of tangency.

\[
(1.5, 3.7)
\]

5. Calculate \( f'(x) \), set it equal to the slope of the secant line and solve the equation.

\[
f'(x) = -2x + 4 \quad -2x + 4 = 1
\]

\[
-2x = -3
\]

\[
x = -3 \div -2 = 1.5
\]

6. Compare your answer to \#5 to your estimated point in \#4, what do you notice?

\[
1.5 \quad \text{same or very close}
\]

Your goal is to see how the MVT actually works and what it means.
The number line has data for the first and second derivative. Use the information to sketch $f(x)$.

$f'(x)$

\begin{align*}
&\text{Neg} & \text{Neg} & \text{Pos} & \text{Pos} \\
&-1 & 0 & 1 \\
&\text{decreasing} & \begin{array}{c}
\text{increasing}
\end{array}
\end{align*}

$f(x)$

\begin{align*}
&\text{cc dn} & \text{cc up} & \text{cc dn} \\
&\text{at } & 1 & 1
\end{align*}

$f''(x)$

\begin{align*}
&\text{Neg} & \text{Pos} & \text{Pos} & \text{Neg} \\
&-1 & 0 & 1 \\
&f(-1) = \frac{3}{4} & f'(0) = 0 & f''(1) = \frac{3}{4}
\end{align*}

Assignment #4: First Derivative and Second Derivative Graphically

1) Page 232 #12, 13, 14, 15, 17.
2) Read p. 234 Example 1. Solve p. 238 #1, 2, 23, 20, 22, 21
1. Verify that the Mean Value Theorem does or does not apply to \( f(x) = x^5 + 2x^3 + 4x - 12 \).

\[
\begin{align*}
  f(-1) &= -1 - 2 - 4 - 12 \\
        &= -19 \\
  f(1) &= 1 + 2 + 4 - 12 \\
        &= -5 \\
\end{align*}
\]

Slope = \[
\frac{-5 - (-19)}{1 - (-1)}
\]

\[
= 7
\]

\[ f'(x) = 5x^4 + 6x^2 + 4 \]

\[ 7 = 5x^4 + 6x^2 + 4 \]

\[ 0 = 5x^4 + 6x^2 - 3 \]  \text{Graph Calc}

\[
x = -0.616, x = 0.616
\]

MVT

2. Find a value \( c \) that satisfies the MVT for \( f(x) = e^{-2x} \) on the interval \([0, 3]\).

\[
\frac{f(3) - f(0)}{3 - 0} = \frac{e^{-6} - 1}{3}
\]

\[
f'(x) = -2e^{-2x}
\]

\[
\frac{e^{-6} - 1}{3} = -2e^{-2x}
\]

\[
\frac{e^{-6} - 1}{-6} = e^{-2x}
\]

\[
\left( \ln \left( \frac{e^{-6} - 1}{-6} \right) = -2x \ln e \right)^{-\frac{1}{2}}
\]

\[
-\frac{1}{2} \ln \left( \frac{e^{-6} - 1}{-6} \right) = x
\]

\[
x \approx 0.897
\]
Goal: To sketch the derivative of a function and correlate where the derivative is positive and negative to where the function is increasing or decreasing.

The graph of \( f(x) \) is shown on the graph below. Sketch the graph of \( f'(x) \).

Give the intervals of which \( f(x) \) is decreasing.

\((-\infty, -4) \cup (-2, 0) \cup (2, \infty)\)

Give the intervals on which the \( f(x) \) is increasing.

\((-4, -2) \cup (0, 2)\)

Give the intervals of which \( f'(x) \) is negative.

\((-\infty, -4) \cup (-2, 0) \cup (2, \infty)\)

Give the intervals on which the \( f'(x) \) is positive.

\((-4, -2) \cup (0, 2)\)

What conclusions can you make from the information above?

when \( f(x) \) is decreasing, \( f'(x) < 0 \)

when \( f(x) \) is increasing, \( f'(x) > 0 \)

How can you use the information above to know if a point is a maximum or a minimum?

If \( f'(x) \) goes from neg to pos, then \( f(x) \) is going from dec to inc, so \( f(x) \) has a min.

If \( f'(x) \) goes from pos to neg, then \( f(x) \) goes from inc to dec so \( f(x) \) has a max.
Topic: Second Derivative

Goal: To sketch the second derivative of a function and correlate where the derivative is positive and negative to where the function is concaved up or concaved down.

The graph at the right shows the function, \( f(x) \), and the first derivative, \( f'(x) \), graphed together.

Sketch the second derivative on the graph below.

On what intervals is the graph of \( f(x) \) concaved up? (smile) concaved down? (frown)
\((-\infty, -3) \cup (-1, 1)\)
\((-3, -1) \cup (1, \infty)\)

On what intervals is the graph of \( f''(x) \) positive? negative?
\((-\infty, -3) \cup (-1, 1)\)
\((-3, -1) \cup (1, \infty)\)

What conclusions can you make from the information above?
\(\text{If } f'' > 0, \text{ then } f \text{ is concave up.}\)
\(\text{If } f'' < 0, \text{ then } f \text{ is concave down.}\)

The number line has the data we collected on the first derivative. Enter the data from the second derivative.

\[
\begin{array}{ccccccc}
\text{\( f'(x) \)} & \text{neg} & \text{pos} & \text{neg} & \text{pos} & \text{neg} \\
\text{\( f(x) \)} & \text{dec} & \text{inc} & \text{dec} & \text{inc} & \text{dec} \\
\text{\( f''(x) \)} & \text{Pos} & \text{Neg} & \text{Pos} & \text{Neg} \\
\text{\( f(x) \)} & \text{cc up} & \text{cc dn} & \text{cc up} & \text{cc dn} \\
\end{array}
\]

On what intervals is the graph of \( f \) both increasing and concaved up? both decreasing and concaved down?
\((-4, -3) \cup (0, 1)\)
\((-2, -1) \cup (2, \infty)\)

The point where a graph goes from concaved down to concaved up or concaved up to concaved down is called a **point of inflection**. Give the \( x \)-coordinates of each point of inflection. How could you find these same \( x \)-values from the second derivative?  
\(x = -3, -1, 1\)

To find a point of inflection, set \( f'' = 0 \)
Complete the conclusion for each interval. Then, graph the function and label everything.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$f''(x)$</th>
<th>Conclusion</th>
</tr>
</thead>
</table>
| $-\infty < x < 1$ | -      | +       |         | $f' < 0 \Rightarrow f \text{ dec}$  
|          |        |         |         | $f'' > 0 \Rightarrow f \text{ cc up}$  |
| $x = 1$  | 0      | undefined | undefined | $(1,0)$ |
| $1 < x < 3$ | +      | -       |         | $f' > 0 \Rightarrow f \text{ inc}$  
|          |         |         |         | $f'' < 0 \Rightarrow f \text{ cc dn}$  |
| $x = 3$  | 4      | 0       | -       | $f' = 0 \Rightarrow f \text{ has a max at } (3,4)$  
|          |         |         |         | $f'' < 0 \Rightarrow f \text{ cc dn}$  |
| $3 < x < 5$ | -      | -       |         | $f' < 0 \Rightarrow f \text{ dec}$  
|          |         |         |         | $f'' < 0 \Rightarrow f \text{ cc dn}$  |
| $x = 5$  | 0      | undefined | undefined | $(5,0)$ |
| $5 < x < \infty$ | +      | +       |         | $f' > 0 \Rightarrow f \text{ inc}$  
|          |         |         |         | $f'' > 0 \Rightarrow f \text{ cc up}$  |

Keep Chart?  
Elim Graph

Assignment #4: Page 232: 13–18 and Page 238: 1, 2, 20–23
If \( f(x) = 5x^2 + x^4 \), find the first and second derivative. Set each derivative equal to zero and solve.

\[
f'(x) = 10x + 4x^3
\]
\[
0 = 10x + 4x^3
\]
\[
0 = 2x(5 + 2x^2)
\]
\[
2x = 0 \quad 5 + 2x^2 = 0
\]
\[
x = 0 \quad 2x^2 = -5
\]
\[
x^2 = -\frac{5}{2}
\]
\[
x = \pm \sqrt{-\frac{5}{2}}
\]
\[\text{not real sol}\]

\[
f''(x) = 10 + 12x^2
\]
\[
0 = 10 + 12x^2
\]
\[
-10 = 12x^2
\]
\[
-\frac{5}{6} = x^2
\]
\[
x = \pm \sqrt{-\frac{5}{6}}
\]
\[\text{no real sol}\]

\( f'' \) will always be greater than 0, so \( f(x) \) is concave up.
**Topic:** Interpreting the Derivatives Graphically

**Goal:** Derive information about the function from the graphs of the derivatives.

The graph at the right is $f'(x)$. Use it to answer the questions.

A. On what intervals is $f(x)$ increasing and decreasing?
   Justify your results.
   - $f(x)$ is inc on $(-\infty, -8) \cup (-4, 0) \cup (3, 2, 5)$. b/c $f'(x) > 0$ on those intervals
   - $f(x)$ is dec on $(-8, -4) \cup (0, 3, 2) \cup (5, \infty)$. b/c $f'(x) < 0$ on those intervals.

B. Give the x–coordinates of any maximum or minimum values of $f$. Justify your results.
   - Max $x = -8, 0, 5$ b/c $f'(x)$ goes from pos to neg $\Rightarrow$ $f(x)$ goes from inc to dec.
   - Min $x = -4, 3, 2$ b/c $f'(x)$ goes from neg to pos $\Rightarrow$ $f(x)$ goes from dec to inc.

C. On what intervals is $f(x)$ concaved up and down? Justify your results.
   - $f'(x)$ is inc on $(-6, -2) \cup (2, 4)$ $\Rightarrow$ $f'' > 0$ on those intervals So $f(x)$ is concave up.
   - $f'(x)$ is dec on $(-\infty, -6) \cup (-2, 2) \cup (4, \infty)$ $\Rightarrow$ $f'' < 0$ on those intervals So $f(x)$ is concave down.

D. Give the x–coordinates of any points of inflection of $f$. Justify your results.
   - $x = -6, -2, 2, 4$ are PII b/c $f'(x)$ goes from dec to inc or inc to dec at those points $\Rightarrow f''$ goes from neg to pos or pos to neg at those points
   - So $f'' = 0$ at $x = -6, -2, 2, 4$

E. Sketch the graph of $f(x)$.

F. Can you apply the Mean Value Theorem to the function $f'(x)$ on the interval $[-8, 4]$? Explain.
   - Yes b/c $f(x)$ is continuous on $[-8, 4]$ and differentiable on $(-8, 4)$

G. Name an interval on which you could apply the mean value theorem with a slope of 0.
   - $[-4, 0]$ b/c $f'(-4) = f'(0) = 0$ or $[-8, -4]$.
The function \( f(x) \) is a piecewise function shown at the right.

A. Find each of the following:
\[
\begin{align*}
f(6) &= 2 \\
\text{slope at } x = 6 &= 2 \\
f'(6) &= 0
\end{align*}
\]

B. What is the area bounded by the graph and the x-axis?
\[
3 + 15 + 3 + 6 = 27 \text{ unit}^2
\]

C. Explain why \( f'(3) \) does not exist.
\[
\lim_{x \to 3^{-}} f'(3) = \lim_{x \to 3^{+}} f'(3)
\]

D. Based upon the graph of \( f \), on what intervals is \( f' \) positive and negative? Justify your reason.
\[
f' > 0 \text{ on } (-4, -2) \cup (5, 7) \text{ b/c } f \text{ is inc on those intervals}
\]
\[
f' < 0 \text{ on } (3, 5) \cup (7, 8) \text{ b/c } f \text{ is dec on those intervals}
\]

E. Graph \( f'(x) \).

F. Write a piecewise function for \( f(x) \) and \( f'(x) \) for \(-4 \leq x \leq 8\).
\[
f(x) = \begin{cases} 
\frac{3}{2}x + 6, & -4 \leq x < 2 \\
3, & 2 \leq x < 3 \\
-\frac{3}{2}x + \frac{15}{2}, & 3 \leq x < 5 \\
2x - 10, & 5 \leq x < 7 \\
-4x + 32, & 7 \leq x \leq 8
\end{cases}
\]
\[
f'(x) = \begin{cases} 
\frac{3}{2}, & -4 \leq x < 2 \\
0, & 2 \leq x < 3 \\
-\frac{3}{2}, & 3 \leq x < 5 \\
2, & 5 \leq x < 7 \\
-4, & 7 \leq x \leq 8
\end{cases}
\]

G. Find the derivative of \( f(x) \), using the piecewise function you found in F. How does the graph of the derivative match your piecewise function of the derivative?