

11.5. GEOMETRIC SERIES

CONSIDER

$$2 + 4 + 8 + 16 + 32 \quad r=2 \quad \text{HAS A SUM}$$

$$2 + 4 + 8 + 16 + 32 + \dots \quad r=2 \quad \text{NO SUM; } \infty$$

$$2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \quad r=\frac{1}{2} \quad \text{HAS A SUM}$$

$$2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \quad r=\frac{1}{2} \quad \text{HAS A SUM}$$

SUM OF A FINITE GEOMETRIC SERIES

$$\text{SUM} = \frac{a_1 (1 - r^n)}{1 - r} \quad r \neq 1$$

SUM OF AN INFINITE GEOMETRIC SERIES

$$\text{SUM} = \frac{a_1}{1 - r} \quad |r| < 1$$

NOTE: $-1 < r < 1$

$$\text{Ex: } \sum_{n=1}^{\infty} 5 \left(\frac{2}{3} \right)^{n-1}$$

$$= \frac{5}{1 - \frac{2}{3}} = \frac{5}{\frac{1}{3}} = 15$$

$$\text{Ex: } \frac{3}{7} + \frac{9}{14} + \frac{27}{28} + \dots$$

No sum as $r = \frac{3}{2} > 1$

$$\text{Ex: } -5 + \frac{10}{3} - \frac{20}{9} + \frac{40}{27} - \frac{80}{81} + \dots$$

$$= \frac{-5}{1 - \left(-\frac{2}{3}\right)} = \frac{-5}{\frac{5}{3}} = -3$$

$r = -\frac{2}{3}$

Ex: FIND THE SUM

$$2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$\begin{aligned} \text{Sum} &= \frac{a_1 (1-r^n)}{1-r} = \frac{2 (1-(\frac{1}{2})^7)}{1-\frac{1}{2}} \\ &= \frac{2 (1-\frac{1}{128})}{\frac{1}{2}} = 4 \left(\frac{127}{128} \right) = \frac{127}{32} \end{aligned}$$

Ex: FIND THE SUM

$$2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$$

$$\text{Sum} = \frac{a_1}{1-r} = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$