11.4 Arithmetic Series

Sequence	Series
$-2, 1, 4, 7, 10, \ldots$	$-2+1+4+7+10+\ldots$

Consider $-7-3+1+5+9+13$

$a_n = a_1 + (n-1)d$

$= -7 + (n-1)4 = -7+4n-4$

$a_n = 4n - 11$

$-7+(-3)+1+5+9+13$ can also be written as

$$\sum_{n=1}^{6} 4n - 11$$

Ex: $\sum_{n=3}^{7} (n-1)^2 + 2 =$

$2^2+2 + 3^2+2 + 4^2+2 + 5^2+2 + 6^2+2 =$

$4+9+16+25+36+10 = 100$

NOW CONSIDER $\sum_{n=1}^{100} n$

$$1 + 2 + 3 + \cdots + 50 + 51 + \cdots + 98 + 99 + 100$$

$$100 + 99 + 98 + \cdots + 51 + 50 + \cdots + 3 + 2 + 1$$

$$101 + 101 + 101 + \cdots + 101 + 101 + \cdots + 101 + 101 + 101$$

THERE ARE 100 OF THESE 101's

$$\text{SUM} = 100\,(101)$$

$$\sum_{n=1}^{100} n = \frac{100\,(101)}{2} = 50\,(101) = 5050$$

✱ SUM OF A FINITE ARITHMETIC SERIES

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$$

$$= \frac{n\,(a_1 + a_n)}{2}$$

Ex: $\sum_{n=1}^{17} (-2n+3)$

$= 1 + (-1) + (-3) + (-5) + \cdots + (-29) + (-31)$

$= \frac{17\left(1 + (-31)\right)}{2} = \frac{17(-30)}{2} = 17(-15)$

$= -255$

Ex: $\sum_{n=4}^{11} \frac{2}{3}n + 1 = \frac{8\left(\frac{11}{3} + \frac{25}{3}\right)}{2}$

$= 4\left(\frac{36}{3}\right) = 48$

Ex: FIND THE SUM

$-17 + (-13) + (-9) + \cdots + 439 + 443$

$a_1 = -17 \quad d = 4 \quad a_n = 443$

$a_n = a_1 + (n-1)d$

$443 = -17 + (n-1)4$

$460 = 4(n-1)$

$115 = n - 1$

$n = 116$

$\text{Sum} = \frac{116(-17+443)}{2} = 24708$