11.3 Geometric Sequences

Consider

$4, 12, 36, 108, 324, \ldots$

$$\frac{a_2}{a_1} = \frac{12}{4} = 3$$

$$\frac{a_3}{a_2} = \frac{36}{12} = 3$$

$$\frac{a_5}{a_4} = \frac{324}{108} = 3$$

} Common Ratio r : 3

Recall:

Arithmetic: $-4, -1, 2, 5, 8, 11, 14, \ldots$ $d = 3$

Geometric: $-3, -4, -\frac{16}{3}, -\frac{64}{9}, -\frac{256}{27}, \ldots$

$r = \frac{4}{3}$

Note: $a_4 = a_3 \cdot r$

$a_4 = a_2 \cdot r \cdot r = a_2 r^2$

$a_4 = a_1 \cdot r \cdot r \cdot r = a_1 r^3$

$a_n = a_1 r^{n-1}$ nth Term

$a_n = a_{n-1} r$ nth Term

Ex: Find the tenth term if $a_6 = 9$ and $a_4 = 4$

$a_6 = a_4 \cdot r^2$

$9 = 4 \cdot r^2$

$r^2 = \frac{9}{4}$

$r = \frac{3}{2}$

$a_{10} = a_6 r^4$

$= 9\left(\frac{3}{2}\right)^4$

$= 9\left(\frac{81}{16}\right)$

$= \frac{729}{16}$

Ex: Find the nth term of

$4, 3, \frac{9}{4}, \frac{27}{16}, \frac{81}{64}, \ldots$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 4 \cdot \left(\frac{3}{4}\right)^{n-1}$

Ex: Find the missing term of the geometric sequence.

$-7, \quad , \frac{-4}{7}, \frac{8}{49}$

$r = \frac{8}{49} \div \frac{-4}{7} = \frac{8}{49} \cdot \left(-\frac{7}{4}\right) = -\frac{2}{7}$

$a_2 = -7\left(-\frac{2}{7}\right) = 2$

$-7, 2, -\frac{4}{7}, \frac{8}{49}$