Guidelines for Graphing Rational Functions

Let $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials with no common factors and $Q(x) \neq 0$.

Step 1) Find and plot the y-intercept (if any) by evaluating $f(0)$.

Step 2) Find the zeros of the numerator (if any) by solving the equation $P(x) = 0$. Then plot the x-intercept(s).

Step 3) Find the zeros of the denominator (if any) by solving the equation $Q(x) = 0$. Then sketch the corresponding vertical asymptotes.

Step 4) Find and sketch the horizontal asymptotes (if any) by using the rule (Horizontal Aysmptotes) given below.

Step 5) Plot *at least* one point between and one point beyond *each* x-intercept and vertical asymptote.

Step 6) Use smooth curves to complete the graph.

Horizontal Asymptotes

Let

$$f(x) = \frac{P(x)}{Q(x)} = \frac{a_n x^n + \cdots}{b_m x^m + \cdots}$$

if $n < m$	the x-axis ($y = 0$) is the Horizontal Asymptote.
if $n = m$	the line $y = \frac{a_n}{b_m}$ is the Horizontal Asymptote.
if $n > m$	there is no Horizontal Asymptote.
if $n = m + 1$	there is a slant asymptote. You must perform division to find it.

These state that if ...

the degree(numerator) $<$ degree(denominator), then the x-axis is the horizontal asymptote.

$$f(x) = \frac{2x - 5}{x^2 - 4}$$

the degree(numerator) $=$ degree(denominator), then the ratio of the leading coefficients give the horizontal asymptote. In this case, $y = \frac{4}{3}$

$$f(x) = \frac{4x^2 - 5}{3x^2 - 4}$$

the degree(numerator) $>$ degree(denominator), then there is no horizontal asymptote.

$$f(x) = \frac{4x^5 - 5}{3x^2 - 4}$$

the degree of the numerator is exactly one more than the degree of the denominator, then there is a slant asymptote and you must perform division to find it.

$$f(x) = \frac{4x^3 - 5}{x^2 - 4}$$

Example: Graph $f(x) = \frac{2(x^2-9)}{x^2-4}$

Step 1) Find and plot the y-intercept (if any) by evaluating $f(0)$.

$$f(0) = \frac{2(0^2-9)}{(0)^2-4} = \frac{2(-9)}{-4} = \frac{-18}{-4} = \frac{9}{2}$$

The y-intercept is $\frac{9}{2}$.

Step 2) Find the zeros of the numerator (if any) by solving the equation $P(x) = 0$. Then plot the x-intercept(s).

$$\begin{aligned} 2(x^2-9) &= 0 \\ x^2-9 &= 0 \\ (x-3)(x+3) &= 0 \end{aligned}$$

The x-intercepts are 3 and -3.

Step 3) Find the zeros of the denominator (if any) by solving the equation $Q(x) = 0$. Then sketch the corresponding vertical asymptotes.

$$\begin{aligned} x^2-4 &= 0 \\ (x-2)(x+2) &= 0 \end{aligned}$$

The vertical asymptotes are 2 and -2.

Step 4) Find and sketch the horizontal asymptotes (if any) by using the rule (Horizontal Aysmptotes) given below.

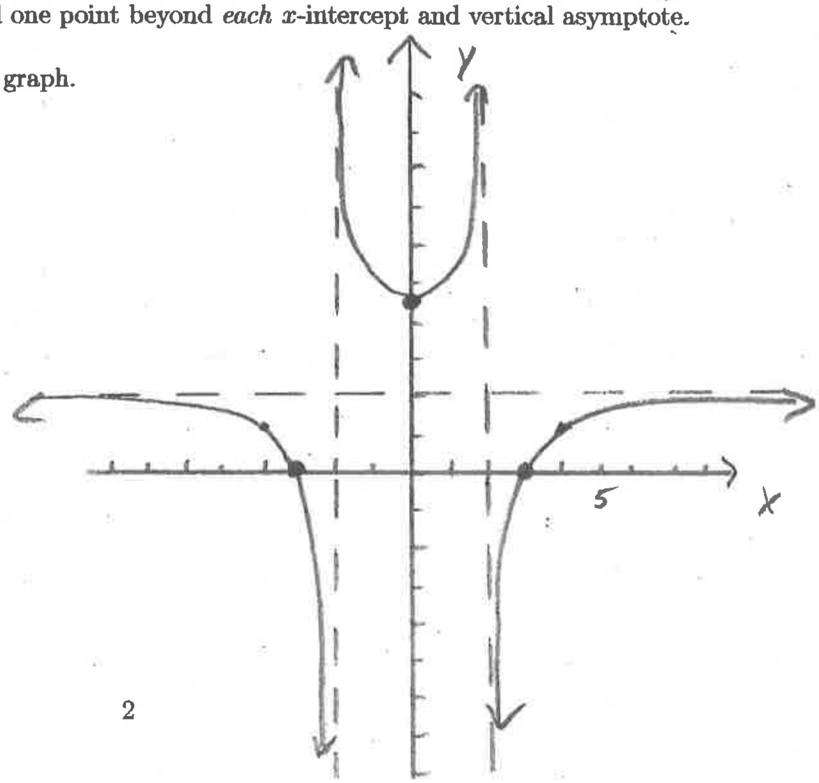
The degree of the numerator, n, is 2.
The degree of the denominator, m, is 2.
Since $n = m$, the line $y = \frac{2}{1}$ is the horizontal asymptote. The 2 and 1 are the leading coefficients of the numerator and denominator, respectively.

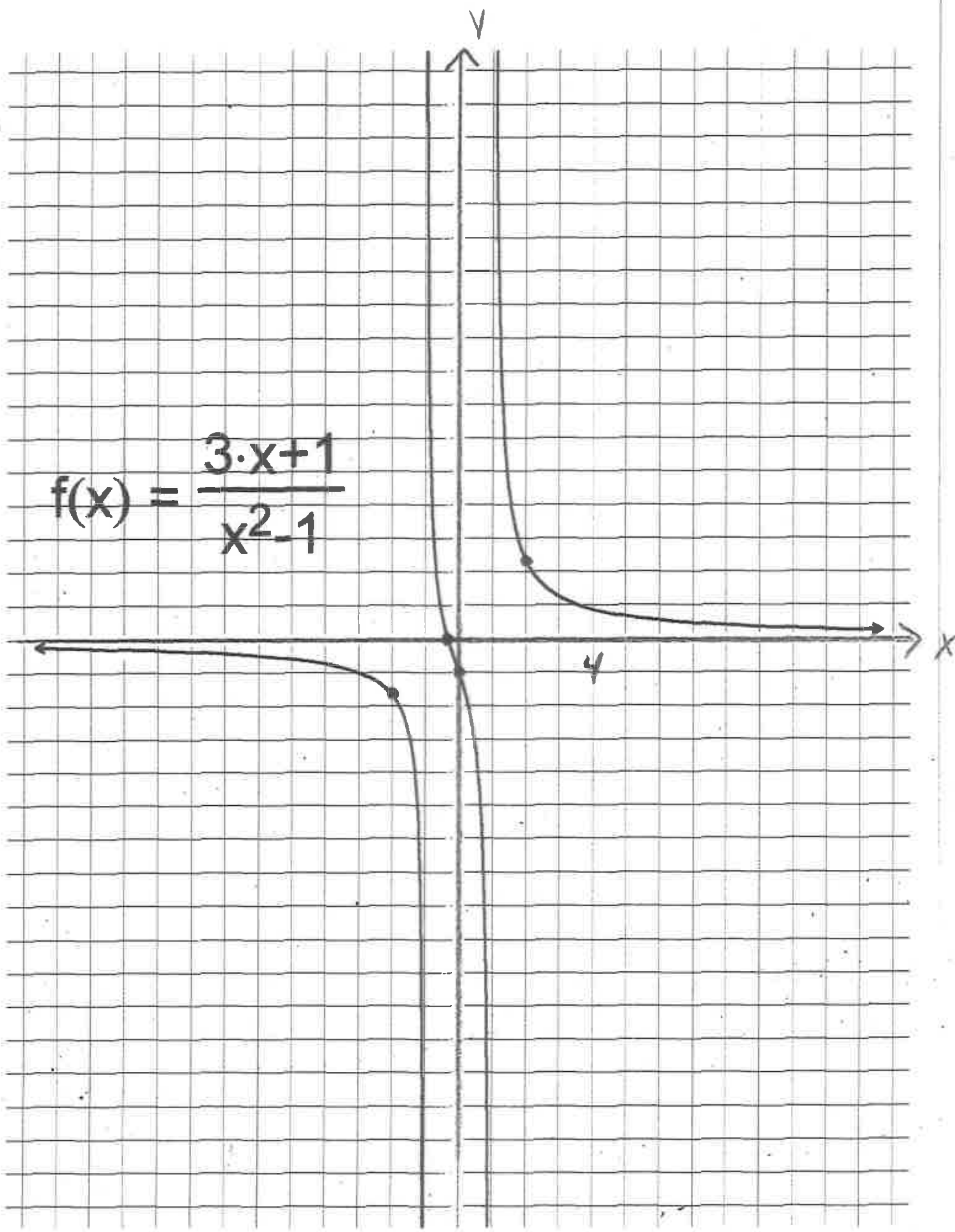
Step 5) Plot *at least* one point between and one point beyond *each* x-intercept and vertical asymptote.

Step 6) Use smooth curves to complete the graph.

X	Y
-4	$\frac{7}{6}$
4	$\frac{7}{6}$

$$\frac{2[(-4)^2-9]}{(-4)^2-4} = \frac{2(7)}{12} = \frac{14}{12} = \frac{7}{6}$$





① Y INTERCEPT :

$$f(0) = \frac{3(0)+1}{0^2-1} = -1$$

② X INTERCEPT : $3X + 1 = 0$

$$X = -\frac{1}{3}$$

③ VERTICAL ASYMPTOTES :

$$X^2 - 1 = 0$$

$$(X-1)(X+1) = 0$$

$$X = 1, -1$$

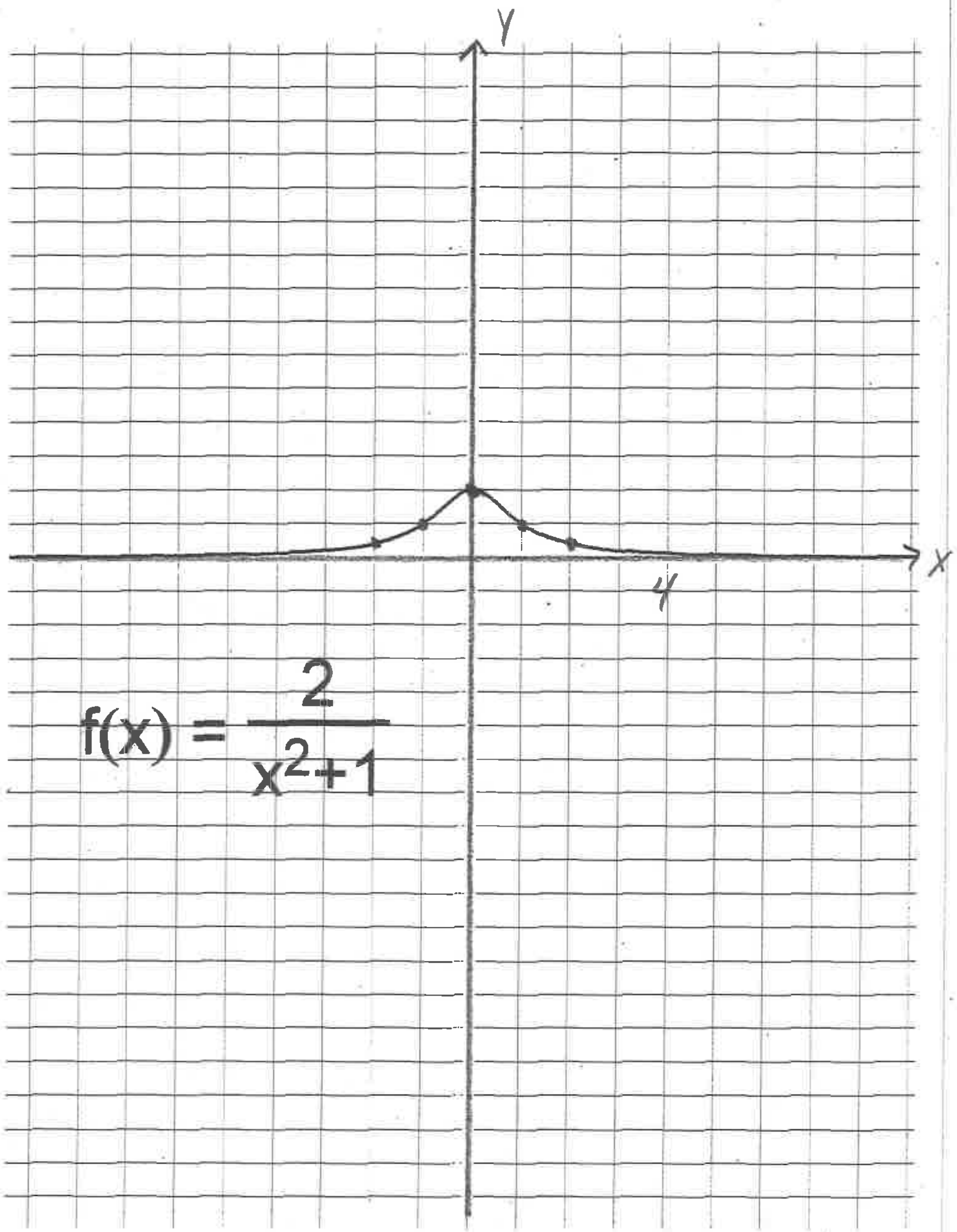
④ HORIZONTAL ASYMPTOTE :

degree (NUMERATOR) $<$ degree (DENOMINAT

X-axis IS ASYMPTOTE

⑤ PLOTTING :

X	Y
2	$7/3$
-2	$-5/3$
$-\frac{1}{2}$	+



$f(x) = \dfrac{2}{x^2+1}$

① Y INTERCEPT:

$$f(0) = \frac{2}{0^2+1} = 2$$

② X INTERCEPT:

$2 = 0$ NONE

③ VERTICAL ASYMPTOTES:

$x^2+1 = 0$

$x^2 = -1$ NONE

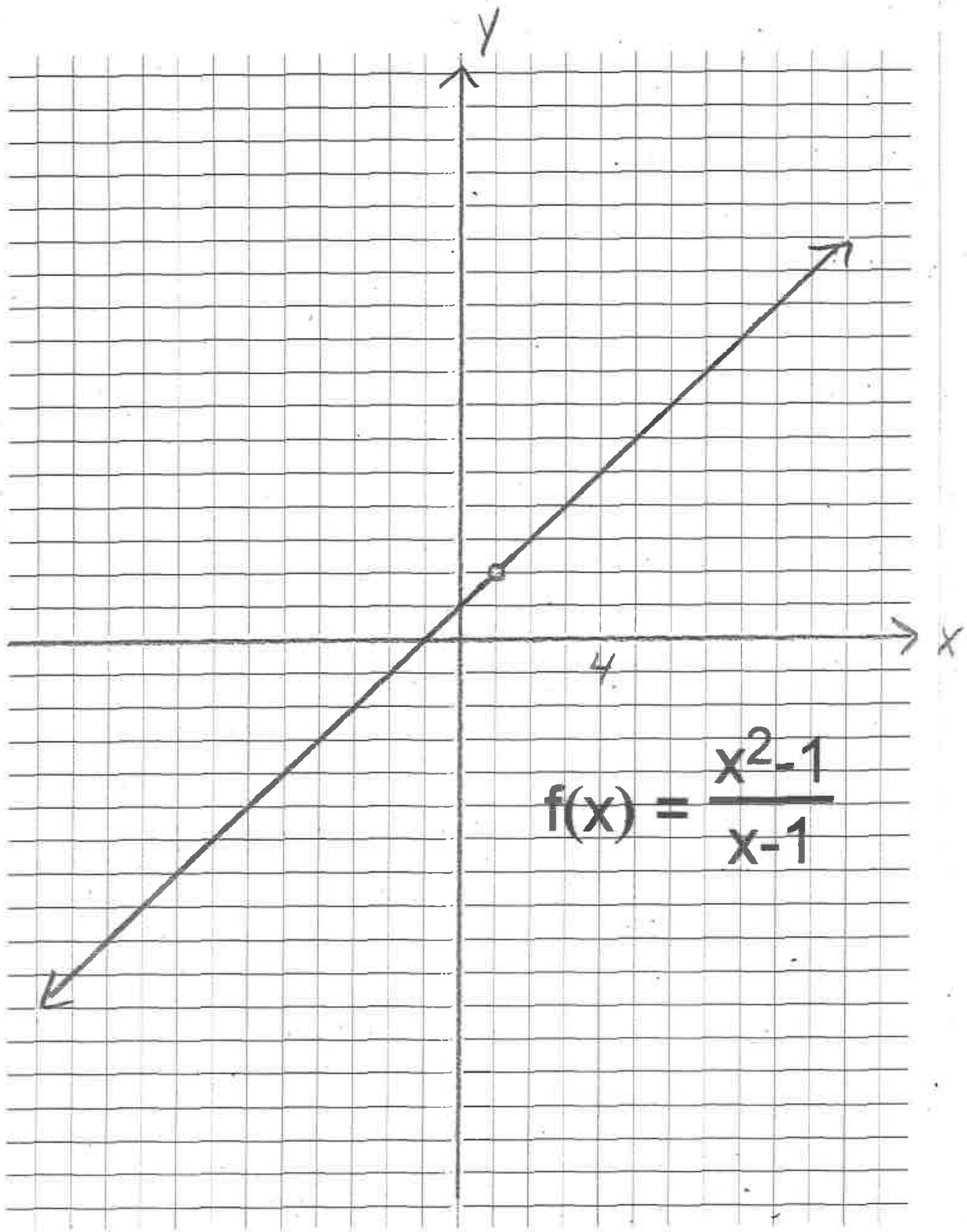
④ HORIZONTAL ASYMPTOTE:

deg (Num) < deg (Den)

X-AXIS IS ASYMPTOTE

⑤ PLOTTING:

X	Y
1	1
-1	1
2	2/5
-2	2/5



NOTE: $f(x) = \dfrac{x^2-1}{x-1}$

$= \dfrac{(x-1)(x+1)}{(x-1)}$

$= x+1$

THIS IS A LINE WITH A "HOLE" AT $x = 1$