7.3 Logarithms

Consider $y = 2^x$

When $x = 3$, $y = 2^3 = 8$ $\quad (3, 8)$

Is there a function that will "undo" the exponential function $y = 2^x$?

Yes! Logarithms

$$y = \log_2 x$$

* If $y = b^x$ then $\log_b y = x$ $\quad (b > 0)$

Ex: $\log_2 8 = 3$ since $2^3 = 8$

$\log_3 81 = 4$ since $3^4 = 81$

$\log_4 \frac{1}{16} = -2$ since $4^{-2} = \frac{1}{16}$

Ex: Find $\log_4 8$

"To what power do we raise 4 to get 8?"

$$4^x = 8$$

$$(2^2)^x = 2^3$$

$$2^{2x} = 2^3$$

$$2x = 3 \qquad x = 3/2$$

Notes: The **Common Logarithm** is the log with base 10.

$\log_{10} x$ can be written $\log x$

The **Natural Logarithm** is the log with base e.

$\log_e x$ can be written $\ln x$.

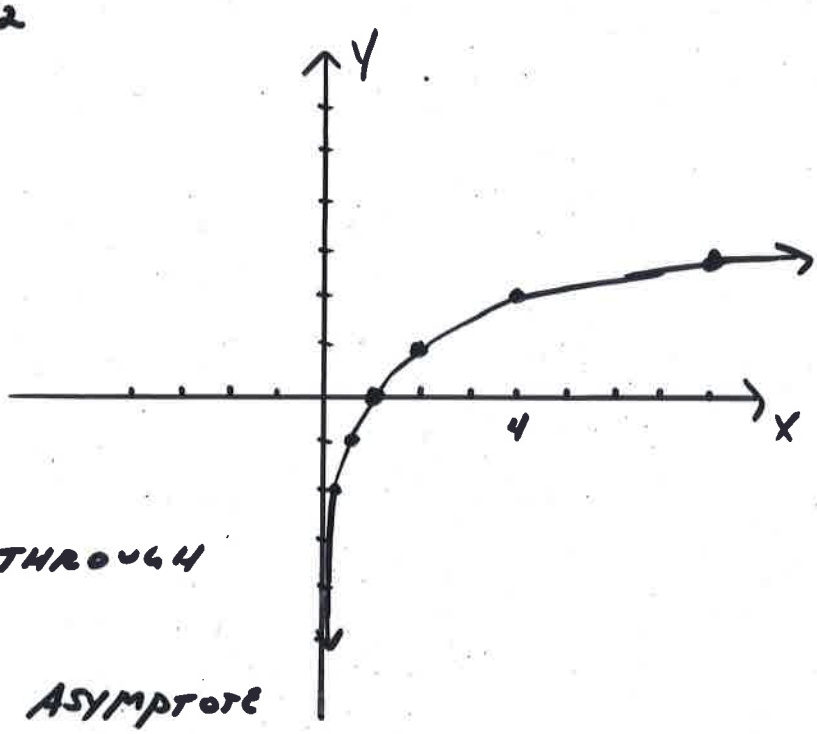
You cannot take the log of a negative number (or 0).

~~$\log_2(-4)$~~ ~~$\log_5(0)$~~ ~~$\log(-2)$~~

Ex: Graph $y = \log_2 x$

x	y
8	3
4	2
2	1
1	0
$\frac{1}{2}$	-1
$\frac{1}{4}$	-2

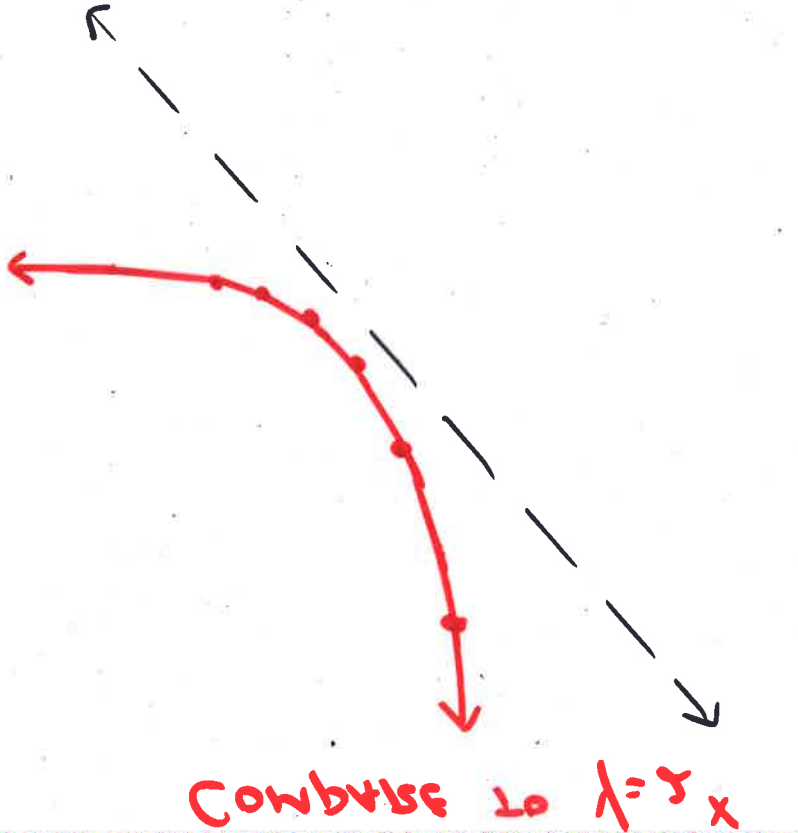
Choose wisely



Note: Passes through (1, 0)

Vertical asymptote at y-axis.

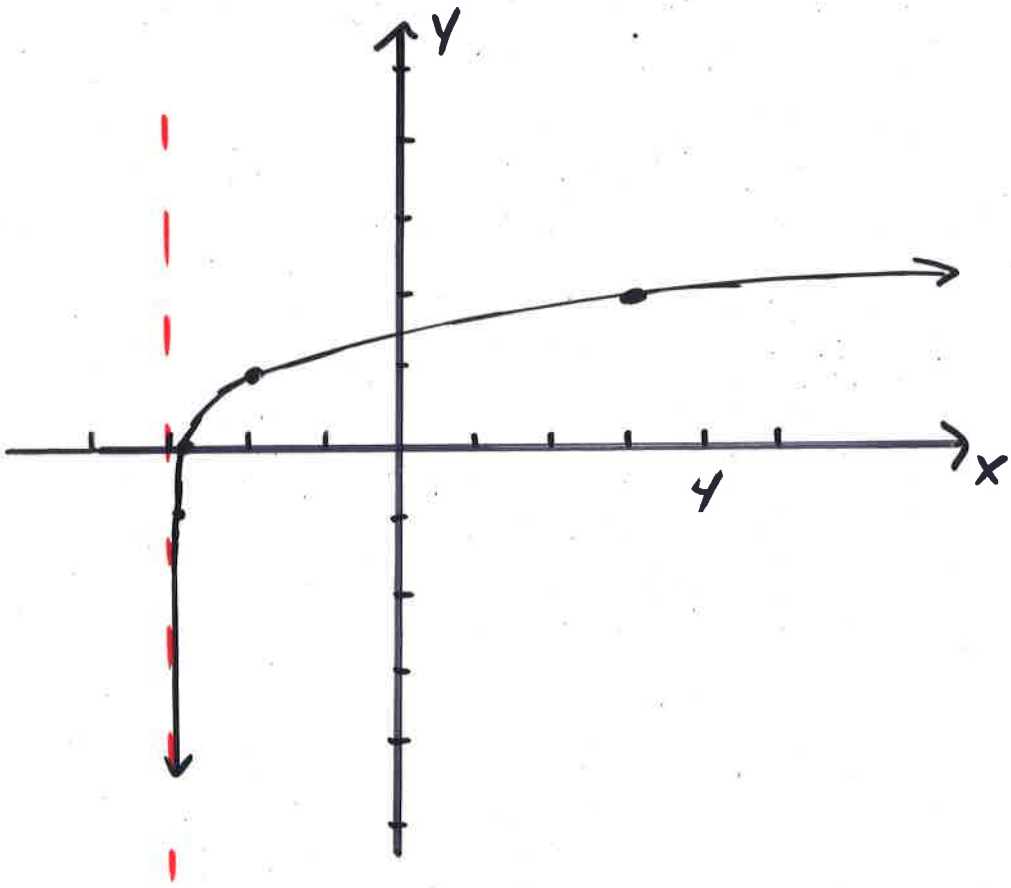
x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



Converse to $y = 2^x$

Ex: Graph $y = \log_6 (x+3) + 1$

x	y
213	4
33	3
3	2
-2	1
$-2\frac{5}{6}$	0
$-2\frac{35}{36}$	-1



It is easy to find

$\log_6 216$ $\log_6 1$

$\log_6 36$ $\log_6 \frac{1}{6}$

$\log_6 6$ $\log_6 \frac{1}{36}$