

6.4 RATIONAL EXPONENTS

RECALL

$$5^2 \cdot 5^4 = 5^{2+4} = 5^6$$

ALSO $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2} + \frac{1}{2}} = 5^1 = 5$

AND $\sqrt{5} \cdot \sqrt{5} = 5$

SINCE $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5$

AND $\sqrt{5} \cdot \sqrt{5} = 5$

THEN $5^{\frac{1}{2}} = \sqrt{5}$

PROPERTY

① $a^{\frac{1}{n}} = \sqrt[n]{a}$

② $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Ex: $16^{\frac{1}{2}} = \sqrt{16} = 4$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

$$\text{Ex: } 4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(\sqrt{4})^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$8^{1.6} = 8^{\frac{5}{3}} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$$

$$(-27)^{\frac{2}{3}} = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$$

$$\left(x^{\frac{2}{3}} y^{-\frac{1}{6}}\right)^{-12} = x^{-8} y^2 = \frac{y^2}{x^8}$$

$$x^{\frac{2}{3}} \cdot x^{\frac{3}{4}} = x^{\frac{2}{3} + \frac{3}{4}} = x^{\frac{4}{4} + \frac{3}{4}} = x^{\frac{7}{4}} = x^{1\frac{3}{4}}$$

$$(-64)^{\frac{1}{2}} \quad \text{NOT REAL}$$

$$(-64)^{\frac{1}{3}} = \sqrt[3]{-64} = -4$$

$$\left(x^{\frac{1}{2}} \cdot x^{\frac{5}{6}}\right)^{\frac{1}{3}} \div x^{\frac{2}{3}} = \frac{\left(x^{\frac{11}{6}}\right)^{\frac{1}{3}}}{x^{\frac{2}{3}}} = \frac{x^{\frac{11}{18}}}{x^{\frac{2}{3}}}$$

$$= x^{\frac{11}{18} - \frac{2}{3}} = x^{\frac{11}{18} - \frac{12}{18}} = x^{-\frac{1}{18}} = \frac{1}{x^{\frac{1}{18}}}$$