# 5-6 The Fundamental Theorem of Algebra

**Essential Understanding** The degree of a polynomial equation tells you how many roots the equation has.

take note

**Theorem** **The Fundamental Theorem of Algebra**

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly $n$ roots, including multiple and complex roots.

take note

**Concept Summary** **The Fundamental Theorem of Algebra**

Here are equivalent ways to state the Fundamental Theorem of Algebra. You can use any one of these statements to prove the others.

- Every polynomial equation of degree $n \geq 1$ has exactly $n$ roots, including multiple and complex roots.
- Every polynomial of degree $n \geq 1$ has $n$ linear factors.
- Every polynomial function of degree $n \geq 1$ has at least one complex zero.

**25.** $y = x^4 + x^3 - 15x^2 - 16x - 16$

Possible Rational Solutions.

$$\frac{\pm 1, 2, 4, 8, 16}{1}$$

~~1~~, 2, 4, 8, 16, ~~-1~~, -2, -4, -8, -16

| | 1 | 1 | -15 | -16 | -16 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | -13 | -29 | -45 |
| -1 | 1 | 0 | -15 | -1 | -15 |
| 2 | 1 | 3 | -9 | -34 | -80 |
| -2 | 1 | -1 | -13 | 10 | -36 |
| 4 | 1 | 5 | 5 | 4 | 0 |

$x^3 + 5x^2 + 5x + 4 = 0$

| | 1 | 5 | 5 | 4 |
|---|---|---|---|---|
| -4 | 1 | 1 | 1 | 0 |

P.R.S. ~~1~~, ~~2~~, ~~4~~, ~~-1~~, ~~-2~~, ~~-4~~

$x^2 + x + 1 = 0$

$x = 4, -4, \frac{-1 \pm i\sqrt{3}}{2}$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{-1 \pm i\sqrt{3}}{2}$$

Things to keep in mind to minimize work

1) Know how many solutions to expect

2) Use Rational Root Theorem to list all possible rational roots.

3) Use Descartes' Rule of Signs to find out how many positive and how many negative solutions are possible.

4) Use upper and lower bounds to eliminate possibilities.