5.5 Theorems About Roots of Polynomial Equations

Rational Root Theorem:

Consider the polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$$

All rational roots will have the form

$$\frac{p}{q}$$

where p is a factor of a_0
and q is a factor of a_n

Ex: List all possible rational solutions of $8x^4 - 7x^3 + 2x^2 - 5x + 6 = 0$

Possible Solutions: $\frac{p}{q} = \pm \frac{1, 2, 3, 6}{1, 2, 4, 8}$

Possible: $\pm 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 2, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, 6$

Ex: Solve $x^3 - 2x^2 - 5x + 10 = 0$

Possible Rational Solutions: $\pm \frac{1, 2, 5, 10}{1}$

$\pm 1, 2, 5, 10$

	1	-2	-5	10	
1	1	-1	-6	4	No
-1	1	-3	-2	12	No
2	1	0	-5	0	Yes

$x^3 - 2x^2 - 5x + 10 = (x-2)(x^2-5)$

Solutions: $x = 2, \pm\sqrt{5}$

Ex: Solve

$$5x^3 - 24x^2 + 41x - 20 = 0$$

Poss. Rat: $\pm \frac{1, 2, 4, 5, 10, 20}{1, 5}$

	5	-24	41	-20
1	5	-19	22	2
-1	5	-29	70	-50
2	5	-14	13	6
-2	5	-34	109	-238
⋮				
$\frac{4}{5}$	5	-20	25	0

$\rightarrow 5x^2 - 20x + 25 = 0$

$5(x^2 - 4x + 5) = 0$

Solve by Q.F.

$x = \frac{4}{5}, 2 \pm i$