

Section 5.4: Dividing Polynomials

- 1) Dividing by a Monomial
- 2) Dividing by Factoring
- 3) Long Division
- 4) Synthetic Division

1) Dividing by a Monomial

$$\frac{12x^4 - 18x^3 + 9x^2 - 4}{4x^2}$$

$$\frac{12x^4}{4x^2} - \frac{18x^3}{4x^2} + \frac{9x^2}{4x^2} - \frac{4}{4x^2}$$

$$3x^2 - \frac{9}{2}x + \frac{9}{4} - \frac{1}{x^2}$$

2) Dividing by Factoring

$$\frac{8x^2 + 2x - 21}{2x - 3}$$

$$\frac{(\cancel{2x-3})(4x+7)}{\cancel{2x-3}}$$

$4x+7$

$$\frac{-168}{2} = -84$$

$4 \cdot -3 = -12$

3) Long Division

$$\begin{array}{r} 4x^3 + 6x^2 - 2x + 8 \\ \hline 2x - 3 \end{array}$$

$$\begin{array}{r} 2x^2 + 6x + 8 \\ 2x - 3 \overline{) 4x^3 + 6x^2 - 2x + 8} \\ \underline{4x^3 - 6x^2} \\ 12x^2 - 2x \\ \underline{12x^2 - 18x} \\ 16x + 8 \\ \underline{16x - 24} \\ 32 \end{array}$$

$$2x^2 + 6x + 8 + \frac{32}{2x - 3}$$

$$(4x^4 - 3x^3 + 5x - 7) \div (x - 2)$$

$$\begin{array}{r}
 4x^3 + 5x^2 + 10x + 25 \\
 x-2 \overline{) 4x^4 - 3x^3 + 0x^2 + 5x - 7} \\
 \underline{4x^4 - 8x^3} \\
 5x^3 + 0x^2 \\
 \underline{5x^3 - 10x^2} \\
 10x^2 + 5x - 7 \\
 \underline{10x^2 - 20x} \\
 25x - 7 \\
 \underline{25x - 50} \\
 43
 \end{array}$$

$$\begin{array}{r}
 4x^3 + 5x^2 + 10x + 25 + 43 \\
 \hline
 x - 2
 \end{array}$$

$$\begin{array}{r}
 25x - 7 \\
 \underline{25x - 50} \\
 43
 \end{array}$$

Ex) DIVIDE

$$(3x^5 - 12x^4 - 9x^3 + 6x - 12) \div (3x^2 + 2x + 1)$$

$$\begin{array}{r}
 \overline{) 3x^5 - 12x^4 - 9x^3 + 0x^2 + 6x - 12} \\
 \underline{3x^5 + 2x^4 + x^3} \\
 -14x^4 - 10x^3 + 0x^2 \\
 \underline{-14x^4 - \frac{28}{3}x^3 - \frac{14}{3}x^2} \\
 -\frac{2}{3}x^3 + \frac{14}{3}x^2 + 6x - 12 \\
 \underline{-\frac{2}{3}x^3 - \frac{4}{9}x^2 - \frac{2}{9}x} \\
 \frac{46}{9}x^2 + \frac{56}{9}x - 12 \\
 \underline{\frac{46}{9}x^2 + \frac{92}{27}x + \frac{46}{27}} \\
 \frac{76}{27}x - \frac{370}{27}
 \end{array}$$

$$x^3 - \frac{14}{3}x^2 - \frac{2}{9}x + \frac{46}{27} + \frac{\frac{76}{27}x - \frac{370}{27}}{3x^2 + 2x + 1}$$

4) Synthetic Division

This shortcut method can be used when dividing by a linear factor $(x-k)$.

Ex: Use Synthetic Division to divide.

$$\begin{array}{r|rrrr} & & x^3 - 14x^2 + 6x - 7 \\ & & x - 2 \\ 2 & 1 & -14 & 6 & -7 \\ & & 2 & -24 & -36 \\ \hline & 1 & -12 & -18 & -43 \end{array}$$

$$x^2 - 12x - 18 - \frac{43}{x-2}$$

Note: If you are missing a power of x in the dividend, then you MUST use 0 in the set-up.

Ex: Use Synthetic Division to divide.

$$\frac{x^5 - 7x^3 - 4x^2 - 7}{x + 3}$$

$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -7 & -4 & 0 & -7 \\ & & -3 & 9 & -6 & 30 & -90 \\ \hline & 1 & -3 & 2 & -10 & 30 & -97 \end{array}$$

$$x^4 - 3x^3 + 2x^2 - 10x + 30 - \frac{97}{x+3}$$

Remainder Theorem: If you divide a polynomial $P(x)$ by $(x - a)$, then the remainder is $P(a)$.

Ex: Use Synthetic Division to find $f(-2)$ where $f(x) = 3x^3 + 5x^2 - 5x + 7$

-2		3	5	-5	7	
			-6	2	6	
		3	-1	-3		
					13	← $f(-2)$