5-3 Solving Polynomial Equations

Objectives To solve polynomial equations by factoring

take note

Concept Summary Polynomial Factoring Techniques

Techniques	Examples
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials For $ax^2 + bx + c$, find factors with product ac and sum b.	$6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$
Difference of Squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$
Factoring by Grouping $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$	$x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$
Sum or Difference of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$

Example: Solve by Factoring

$$8x^3 = 27$$

$8x^3 - 27 = 0$

$(2x)^3 - (3)^3 = 0$

Formula for Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$8x^3 - 27 = (2x-3)(4x^2+6x+9) = 0$

$2x - 3 = 0$ or $4x^2 + 6x + 9 = 0$

$x = \frac{3}{2}$

$x = \frac{-6 \pm \sqrt{36 - 4(4)(9)}}{2(4)}$

$= \frac{-6 \pm \sqrt{-108}}{8}$

$= \frac{-6 \pm i\sqrt{108}}{8}$

$= \frac{-6 \pm 6i\sqrt{3}}{8}$

$= \frac{-3 \pm 3i\sqrt{3}}{4}$

$x = \frac{3}{2}, \frac{-3 \pm 3i\sqrt{3}}{4}$

Example: Solve by Factoring

$$27 = -x^4 - 12x^2$$

This is "Quadratic in Form". Although it is not quadratic, it can be solved using quadratic techniques.

$$x^4 + 12x^2 + 27 = 0$$

$$(x^2 + 9)(x^2 + 3) = 0$$

$$x^2 = -9 \qquad x^2 = -3$$

$$x = \pm\sqrt{-9} \qquad x = \pm\sqrt{-3}$$

$$x = \pm 3i \;,\; \pm i\sqrt{3}$$

Example: Solve by Factoring

$$x^3 - 2x + 1 = 4x^2 + x - 11$$

With 4 terms, consider "Factor by Grouping"

$$x^3 - 4x^2 - 3x + 12 = 0$$

$$(x^3 - 4x^2) - (3x - 12) = 0$$

NOTE THE SIGN CHANGE

$$x^2(x-4) - 3(x-4) = 0$$

$$(x-4)(x^2-3) = 0$$

$x - 4 = 0$ $\qquad$ $x^2 - 3 = 0$

$x^2 = 3$

$x = 4, \pm\sqrt{3}$