4.8 Complex Numbers

Ex: Solve $x^2 + 1 = 0$

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

No Real Solution

Defn: $\sqrt{-1} = i$

Ex: Simplify

a) $\sqrt{-3}$

b) $\sqrt{-4}$

c) $\sqrt{-32}$

a) $\sqrt{-3} = \sqrt{3}\sqrt{-1} = \sqrt{3} \cdot i = i\sqrt{3}$

b) $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = 2 \cdot i = 2i$

c) $\sqrt{-32} = \sqrt{16} \cdot \sqrt{-1} \cdot \sqrt{2} = 4i\sqrt{2}$

NOTE: $i^2 = (\sqrt{-1})^2 = -1$

COMPLEX NUMBERS

$$a + bi$$

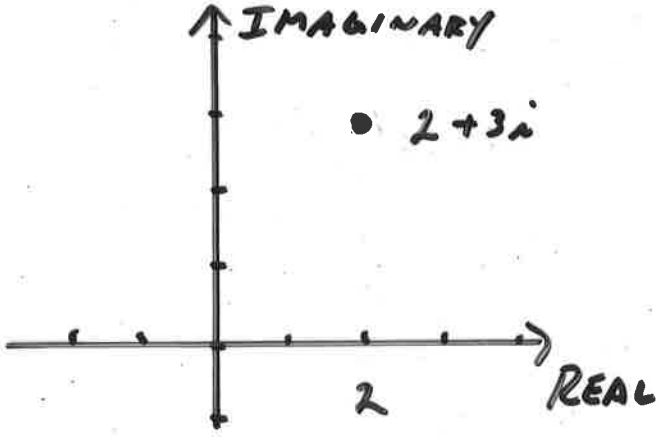
a: REAL PART

bi: IMAGINARY PART

Ex: $2 + 3i$, $4 - \frac{4}{3}i$, $5 + 3i\sqrt{2}$

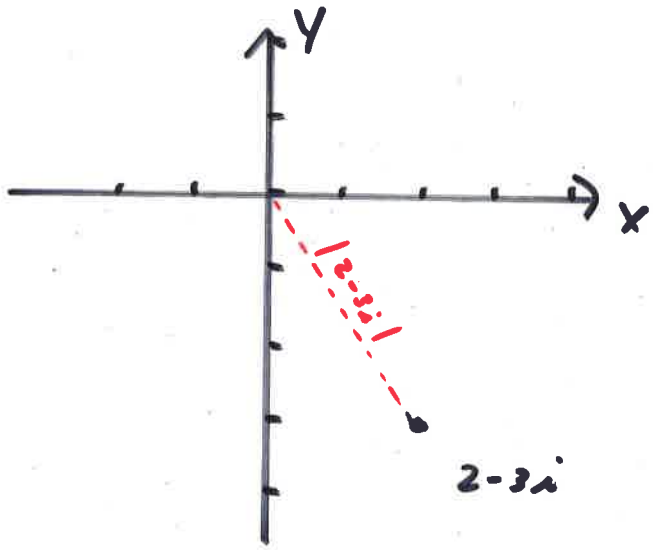
4, $-2i$ ARE ALL COMPLEX NUMBERS

Ex: GRAPH $2 + 3i$ ON THE COMPLEX NUMBER PLANE



NOTE: THE ABSOLUTE VALUE OF A COMPLEX NUMBER IS THE DISTANCE FROM THE ORIGIN

Ex: Find $|2-3i|$



* Note: $|a+bi| = \sqrt{a^2+b^2}$

$|2-3i| = \sqrt{2^2+3^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$\rightarrow \sqrt{2^2+(-3)^2}$

Ex: $|2i| = \sqrt{0^2+2^2} = \sqrt{4} = 2$

* To Add/Subtract, Combine Real Parts and Imaginary Parts

Ex: $(2-3i) - (4-2i)$

$2-3i-4+2i$

$-2-i$

* To Multiply, treat like binomials and FOIL

Ex: $(2-3i)(4-2i)$

$8-4i-12i+6i^2$

$= 8-16i+6(-1) = 8-16i-6$

$= 2-16i$

To Divide Complex Numbers,

Multiply the Numerator and the Denominator by the Conjugate of the Denominator.

Ex: $\dfrac{2+3i}{1+4i}$

$$\frac{2+3i}{1+4i} \cdot \frac{1-4i}{1-4i} = \frac{2-8i+3i-12i^2}{1-4i+4i-16i^2}$$

$$= \frac{14-5i}{1+16} = \frac{14-5i}{17}$$

Ex: Solve $4x^2+1=0$

$$4x^2 = -1$$

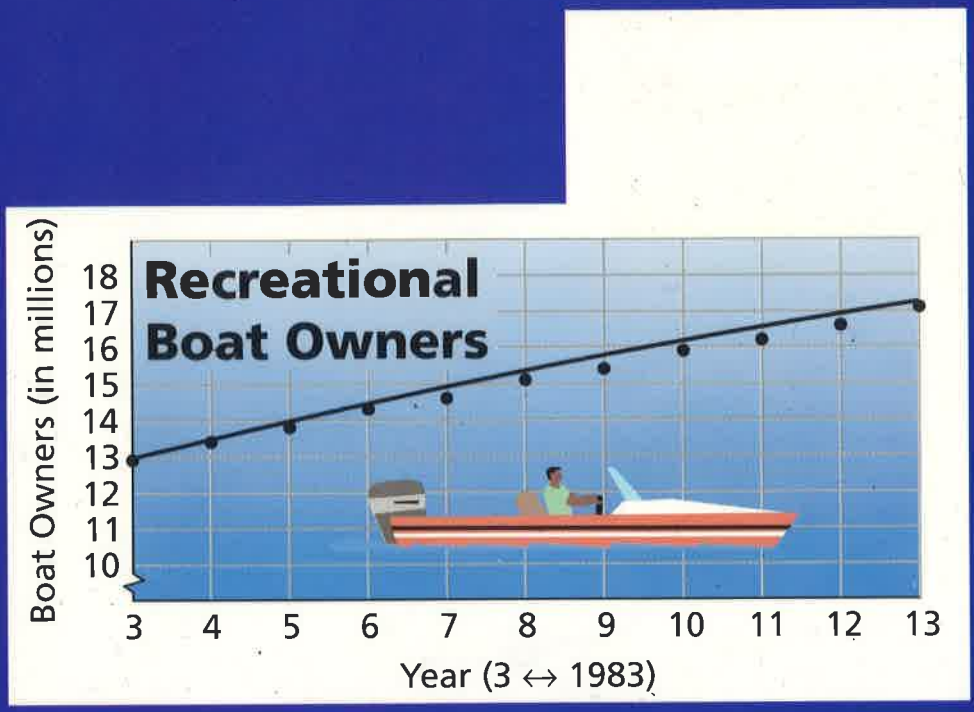
$$x^2 = -\frac{1}{4}$$

$$x = \pm\sqrt{-\frac{1}{4}} = \pm\sqrt{-1}\sqrt{\frac{1}{4}} = \pm i\left(\frac{1}{2}\right)$$

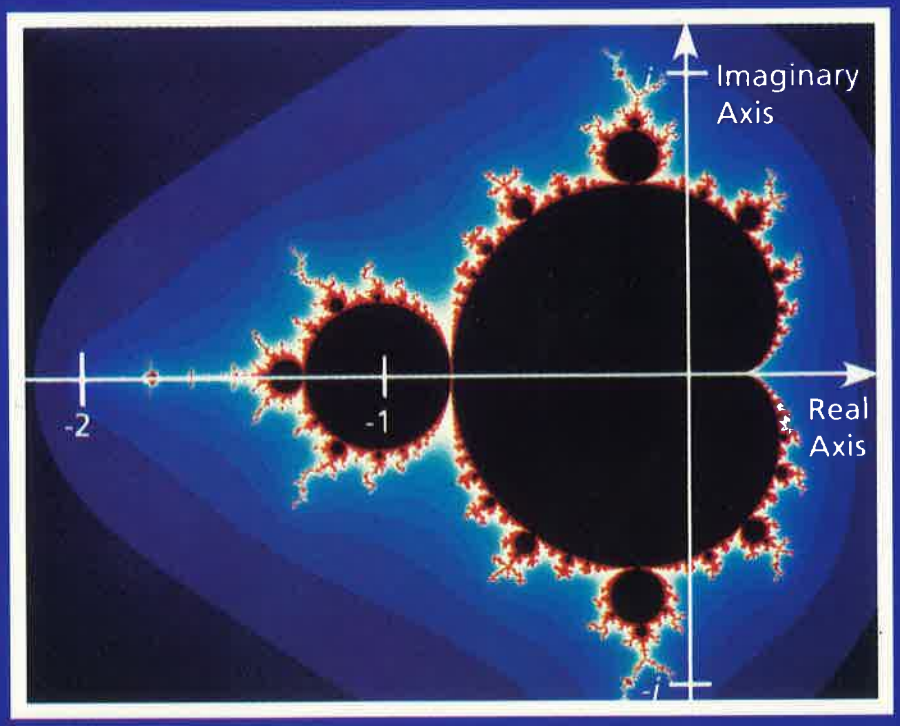
$$= \pm\frac{i}{2}$$

HW:

Lesson 5.4, Page 256, Exercise 44

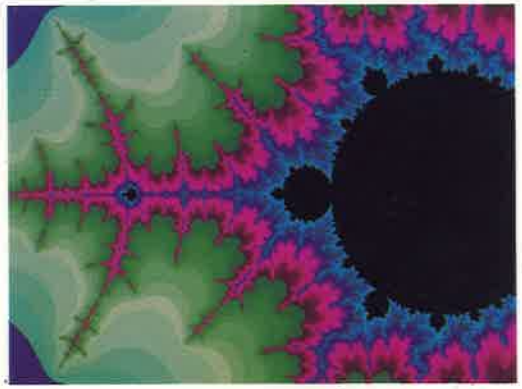


Lesson 5.5, Page 261, Example 5

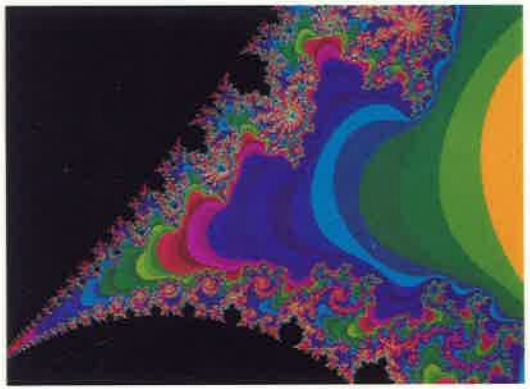


Lesson 5.5, Page 263, Exercises 69-72

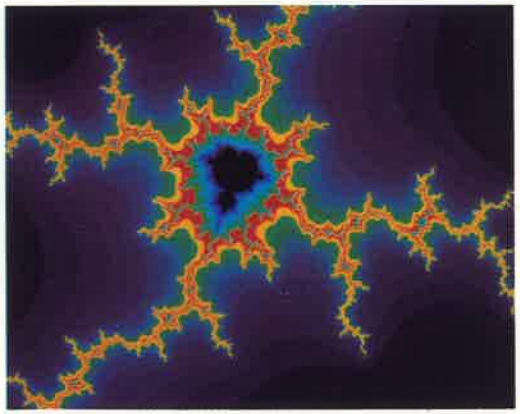
69. *Cos Antenna* $(6.2 - 1.2i) - (7.6 - 1.2i)$



70. *Star Factory* $-\frac{1}{170}(3 + 5i)(8 + 15i)$



71. *Lighting* $(1 + 0.5i)(0.396 + 0.822i)$



72. *Spanish Lace* $2i\,(0.1 - 0.36i)$