3.5 Solving Systems of Equations in 3 Variables

Objective: To solve a 3 x 3 system of equations

Ex: Solve

$$5x - 2y + 8z = -4 \quad (1)$$
$$x + 3y - 6z = 9 \quad (2)$$
$$2x + y + 4z = 6 \quad (3)$$

First, eliminate the SAME variable from two equations

$-5(2) + (1)$

$$\begin{array}{r} -5x - 15y + 30z = -45 \\ 5x - 2y + 8z = -4 \\ \hline -17y + 38z = -49 \end{array}$$

$-2(2) + (3)$

$$\begin{array}{r} -2x - 6y + 12z = -18 \\ 2x + y + 4z = 6 \\ \hline -5y + 16z = -12 \end{array}$$

Solve the resulting equations as usual.

$$-17Y + 38Z = -49 \quad (4)$$

$$-5Y + 16Z = -12 \quad (5)$$

$5 \cdot (4)$ $$-85Y + 190Z = -245$$

$-17 \cdot (5)$ $$85Y - 272Z = 204$$

$$-82Z = -41$$

$$Z = \frac{1}{2}$$

"BACK-SUBSTITUTE" INTO (5)

$$-5Y + 16\left(\frac{1}{2}\right) = -12$$

$$-5Y + 8 = -12$$

$$-5Y = -20$$

$$Y = 4$$

NOW INTO (2)

$$X + 3Y - 6Z = 9$$

$$X + 3(4) - 6\left(\frac{1}{2}\right) = 9$$

$$X + 12 - 3 = 9$$

$$X + 9 = 9$$

$$X = 0$$

SOLUTION: $\left(0, 4, \frac{1}{2}\right)$

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Example: Solve the system of equations given below.

$$\begin{aligned} 4x - 2y - 3z &= 5 \qquad (1) \\ -8x - y + z &= -5 \qquad (2) \\ 2x + y + 2z &= 5 \qquad (3) \end{aligned}$$

1. Choose one variable to eliminate.

2. Then select two of the three equations and work to get one equation in two variables.

3. Next, use a different pair of equations and eliminate the same variable that you did in Step 2.

4. Solve the resulting system of equations (2 equations, 2 variables).

5. Substitute the solution from Step 4 into one of the original three equations and solve for the third variable.

$$\begin{aligned} -8x - y + z &= -5 \\ 2x + y + 2z &= 5 \\ \hline -6x \quad + 3z &= 0 \end{aligned} \qquad (4)$$

$$\begin{aligned} 4x - 2y - 3z &= 5 \\ 4x + 2y + 4x &= 10 \\ \hline 8x \quad + z &= 15 \end{aligned} \qquad (5)$$

Now we solve the resulting system of equations (4) and (5).

$$\begin{aligned} -6x + 3z &= 0 \\ 8x + z &= 15 \end{aligned}$$

Multiplying equation 5 by -3, we get:

$$\begin{aligned} -6x + 3z &= 0 \\ -24x - 3z &= -45 \end{aligned}$$

This leads to...

$$\begin{aligned} -30x &= -45 \\ x &= \frac{-45}{-30} = \frac{3}{2} \end{aligned}$$

We use equation 4 to get

$$\begin{aligned} -6x + 3z &= 0 \\ -6 \cdot \frac{3}{2} + 3z &= 0 \\ -9 + 3z &= 0 \\ z &= 3 \end{aligned}$$

Finally, we use any of the original equations and substitute to find the third variable, y. Let's choose equation 3.

$$\begin{aligned} 2x + y + 2z &= 5 \\ 2 \cdot \frac{3}{2} + y + 2 \cdot 3 &= 5 \\ 3 + y + 6 &= 5 \\ y &= -4 \end{aligned}$$

The solution is the ordered triple $\left(\frac{3}{2}, -4, 3\right)$.

Application: A technique can be used to find the equation of the parabola through three points. This equation can be used to find the location of any object following a parabolic path. Examples of such objects are: certain comets, thrown ball, fired projectile, etc.

All quadratic equations (the graph is a parabola) can be written in the form

$$y = ax^2 + bx + c$$

where $a, b,$ and c are constants.

Example: Find the equation of the parabola passing through the points $(-2, 3)$, $(1, 1)$, and $(0, 3)$.

By plugging in each ordered pair into the equation above, we get three separate equations.

By plugging in $(-2, 3)$, we get

$$\begin{aligned} 3 &= a(-2)^2 + b(-2) + c \\ 3 &= 4a - 2b + c \qquad (6) \end{aligned}$$

By plugging in $(1, 1)$, we get

$$\begin{aligned} 1 &= a(1)^2 + b(1) + c \\ 1 &= a + b + c \qquad (7) \end{aligned}$$

By plugging in $(0, 3)$, we get

$$\begin{aligned} 3 &= a(0)^2 + b(0) + c \\ 3 &= c \qquad (8) \end{aligned}$$

Now considering equations (6) , (7), and (8), we get the system of equations below.

$$\begin{aligned} 4a - 2b + c &= 3 \\ a + b + c &= 1 \\ c &= 3 \end{aligned}$$

When solved, the solution is $\left(-\frac{2}{3}, -\frac{4}{3}, 3\right)$. This gives us the requested quadratic equation. The equation is

$$y = -\frac{2}{3}x^2 - \frac{4}{3}x + 3$$

This equation can now be used to find infinitely many additional points. If the speed of the object is known, the equation can be used to find the location of the object and any given time.