# 1. Section 3.4 Linear Programming

**Objective: How to solve a linear programming problem.**

The look of an example:

Objective Function

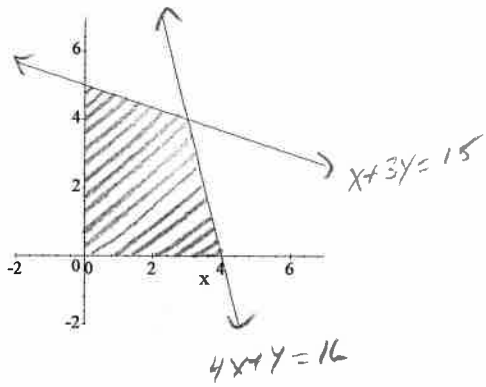
$$f(x,y) = 3x + 2y$$

this is the function we will want to maximize or minimize.

Subject to:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + 3y &\leq 15 \\ 4x + y &\leq 16 \end{aligned}$$

the above function has these requirements that must be met.




*Guidelines for Solving a Linear Programming Problem*

Step 1. Sketch the region corresponding to the system of constraints.

Step 2. Find the vertices of the **Feasible Region**.

Step 3. Test the **Objective Function** at each of the vertices and select the values of the variables that optimize the objective function. For a **bounded region**, both a minimum and a maximum will exist. For an **unbounded region**, *if* an optimal solution exists, it will occur at a vertex.

Example: The AC Telephone Company manufactures two styles of cordless telephones, deluxe and standard. Each deluxe telephone nets the company \$9 in profit, and each standard telephone nets \$6 in profit. Machines A and B are used to make both styles of telephones. Each deluxe telephone requires three hours of machine A time and one hour of machine B time. Each standard telephone requires two hours of machine A time and two hours of machine B time. An employee has an idea that frees twelve hours of machine A time and eight hours of machine B time. Determine the mix of telephones that can be made during the free time that most effectively generates profit for the company within the given constraints.

Define Variables

Let $d$ be the number of deluxe telephones.

Let $s$ be the number of standard telephones.

Write out problem. Since the profit on each deluxe phone is \$9 and the profit on each standard phone is \$6, the profit function is $P(d,s) = 9d + 6s$.

Maximize $P(d,s) = 9d + 6s$

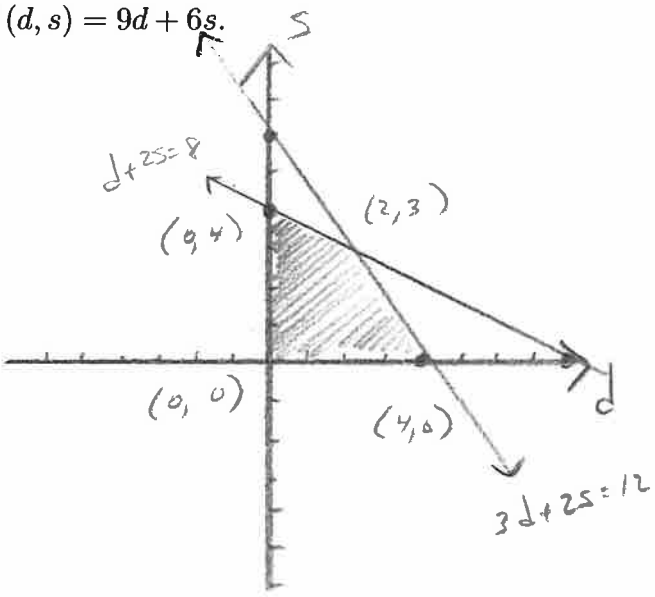
Subject to:

$d \geq 0$

$s \geq 0$

$3d + 2s \leq 12$

$d + 2s \leq 8$




$P(0,0) = 9(0) + 6(0) = 0$

$P(0,4) = 9(0) + 6(4) = 24$

$P(2,3) = 9(2) + 6(3) = 36$

$P(4,0) = 9(4) + 6(0) = 36$

This problem has alternate optimal solutions. The company will make the same profit it they make and sell 2 deluxe telephones and 3 standard telephones as it will from making and selling 4 deluxe telephones and no standard telephones.