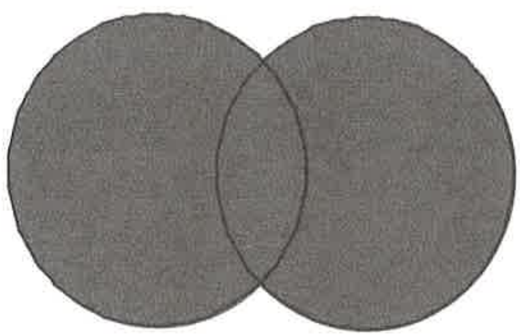
Section 11.3 Probability of Multiple Events

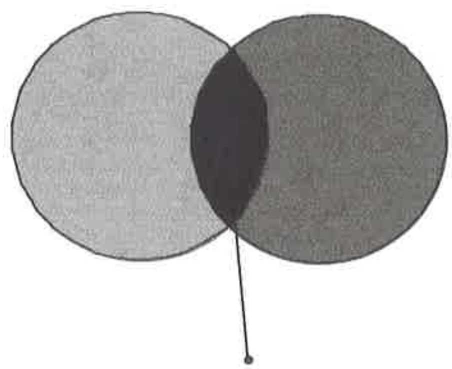
Definition 1 *The union of two events, A and B, written as $A \cup B$, consists of all outcomes that are in A or in B (or in both A and B).*

Definition 2 *The intersection of A and B, written as $A \cap B$, consists of all outcomes that are in A and B.*

Consider the following Diagrams.



$A \cup B$

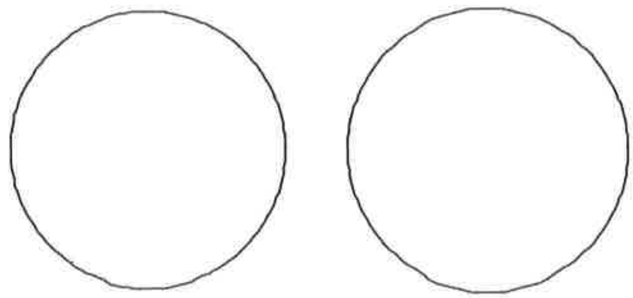


$A \cap B$

For example, consider the two sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$.

$$\begin{aligned} A \cup B &= \{1, 2, 3, 4, 5\} \\ A \cap B &= \{3\} \end{aligned}$$

Note: If $A \cap B = \{\}$, then A and B are said to be mutually exclusive. In other words, if A and B have nothing in common, then they are mutually exclusive.



Two mutually exclusive events

Theorem 3 *If A and B are events, then the probability of A or B occurring is*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note: If A and B are mutually exclusive, then the above theorem becomes

$$P(A \cup B) = P(A) + P(B)$$

Example 4 *What is the probability of drawing a king or a black card from a standard deck of cards?*

It is possible to draw a card that is both a king and a black card ($K♣, K♠$). Therefore, these events are not mutually exclusive. So we get...

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(\text{King or black}) &= P(\text{King}) + P(\text{black}) - P(\text{King and black}) \\ &= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13} \end{aligned}$$

Example 5 *Johnny has 4 pennies, 3 nickels, and 6 dimes in his pocket. He takes one coin from his pocket at random. What is the probability that it is a penny or a dime?*

These are mutually exclusive events since a coin cannot be a penny and a dime.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ P(\text{penny or a dime}) &= P(\text{penny}) + P(\text{dime}) \\ P(\text{penny or a dime}) &= \frac{4}{13} + \frac{6}{13} = \frac{10}{13} \end{aligned}$$

Definition 6 *Two events are independent if the occurrence of one event has no effect on the occurrence of the other event.*

For example, if a coin is tossed two times, the result of the second toss has nothing to do with the result of the first toss. The two tosses are independent.

Definition 7 *Two events are dependent if the occurrence of one event* <u>does</u> *affect the occurrence of the other.*

For example, if two cards are drawn from a standard deck (without replacement), the probability that the second card is an Ace depends upon what the first drawn card was. The two draws are dependent.

Theorem 8 *If A and B are independent events, then the probability that both A and B occurs is*

$$P(A \cap B) = P(A) \cdot P(B)$$

Example 9 *Find the probability of getting a sum of 7 on the first toss of two dice and a sum of 4 on the second toss.*

Let A be "a sum of 7 on the first toss".
Let B be "a sum of 4 on the second toss".

Then

$$P(A) = \frac{6}{36} \text{ and } P(B) = \frac{3}{36}$$

The two tosses are independent.

$$\begin{aligned} P(A \text{ and } B) &= P(A) \cdot P(B) \\ &= \frac{6}{36} \cdot \frac{3}{36} = \frac{1}{72} \end{aligned}$$

The probability of a 7 on the first toss AND a 4 on the second toss is $\frac{1}{72}$.

Example 10 *A box contains 30 pieces of paper numbered from 1 to 30. Three pieces are drawn from the box (without replacement). Find the probability that all three pieces have numbers less than 10.*

After the first piece of paper is selected, there are only 29 pieces of paper left for the second choice to be made. Once the second piece has been selected, there are only 28 pieces remaining for the final piece to be selected. These events are dependent. We get...

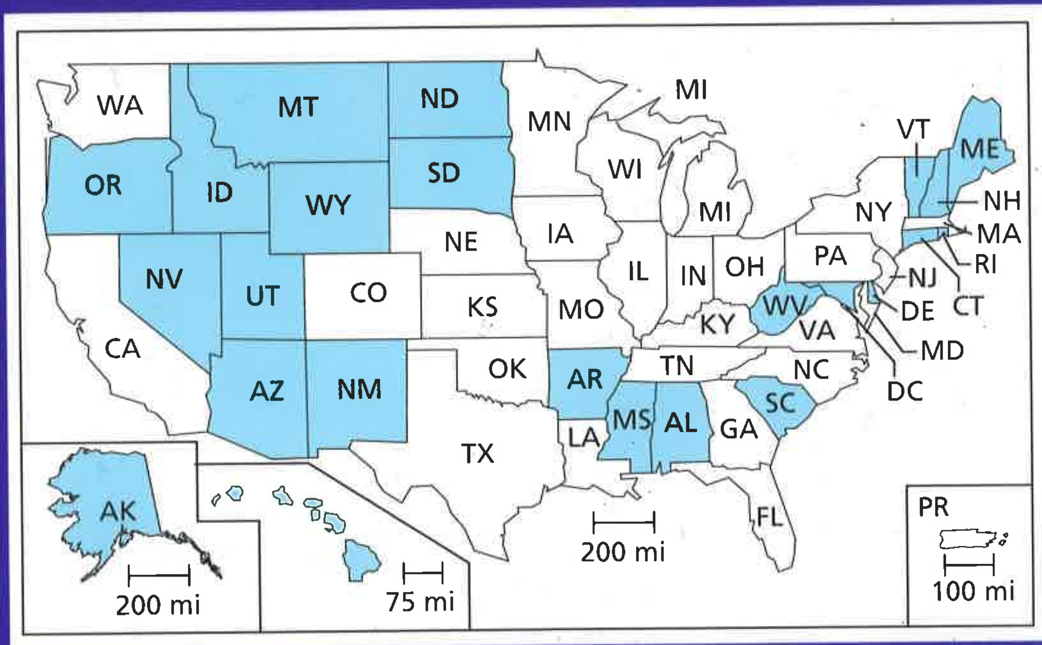
$$\begin{aligned} P\left(\text{selecting 3 pieces that all have numbers less than 10}\right) &= \frac{9}{30}\cdot\frac{8}{29}\cdot\frac{7}{28} \\ &= \frac{3}{145} \end{aligned}$$

Example 11 *Consider a family that has 4 children. All 4 are boys. The mother is pregnant with their 5th child. What is the probability that the child will be a girl?*

The outcome of the next child does not depend on the outcome of the previous four births. This is an independent event. There are two possible outcomes (boy or girl), each of which is equally likely.

$$P\left(\text{next child is a girl}\right) = \frac{1}{2}$$

Lesson 15.2, Page 789, Example 3



Lesson 15.2, Page 791, Communicating about Algebra