11-10 Normal Distributions

Common Core State Standards

S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate . . .

MP 1, MP 3, MP 4

Objective To use a normal distribution

Try it out. Suppose (2, 2) is a point on f(x). If f(x) is even, what other point is on the graph of f(x)?

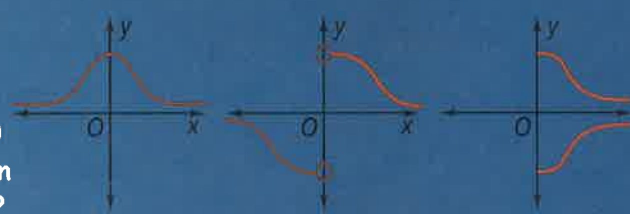
SOLVE IT! Getting Ready!

Even and odd functions are defined as follows.

Even function: $f(x) = f(-x)$

Odd function: $-f(x) = f(-x)$

Which is the graph of an even function? Of an odd function? Justify your answers.



MATHEMATICAL PRACTICES A **discrete probability distribution** has a finite number of possible events, or values. The binomial probability distribution you studied in the preceding lesson is a discrete probability distribution.

The events for a **continuous probability distribution** can be any value in an interval of real numbers. If a data set is large, the distribution of its discrete values approximates a continuous distribution.

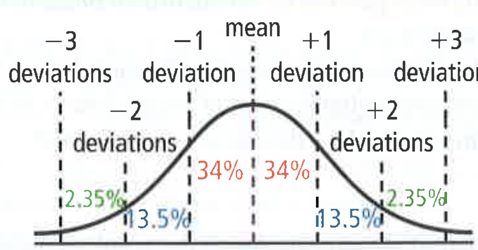
Lesson Vocabulary
- discrete probability distribution
- continuous probability distribution
- normal distribution

Essential Understanding Many common statistics (such as human height, weight, or blood pressure) gathered from samples in the natural world tend to have a *normal distribution* about their mean.

A **normal distribution** has data that vary randomly from the mean. The graph of a normal distribution is a normal curve.

take note

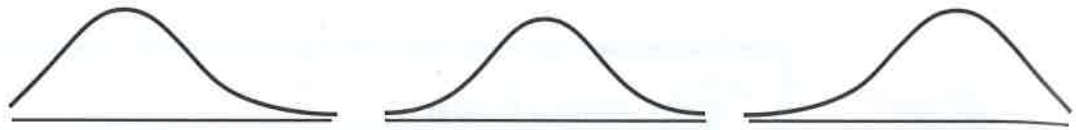
Key Concept Normal Distribution



In a normal distribution,
- 68% of data fall within one standard deviation of the mean
- 95% of data fall within two standard deviations of the mean
- 99.7% of data fall within three standard deviations of the mean

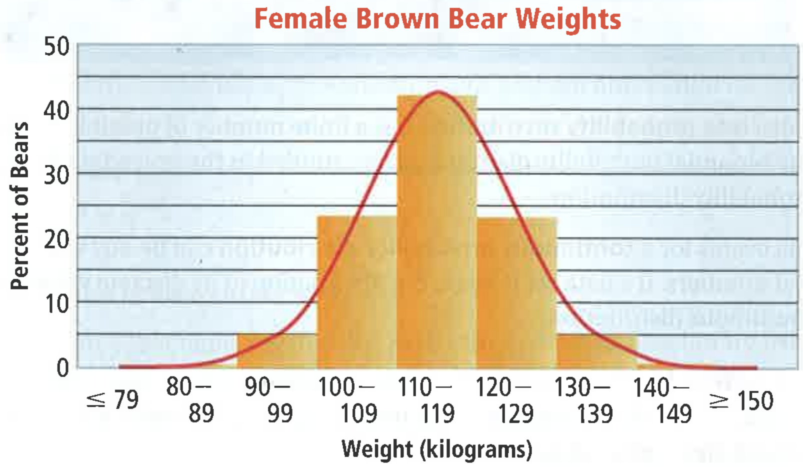
A normal distribution has a symmetric bell shape centered on the mean.

Sometimes data are not normally distributed. A data set could have a distribution that is *skewed*, an asymmetric curve where one end stretches out further than the other end. When a data set is skewed, the data do not vary predictably from the mean. This means that the data do not fall within the standard deviations of the mean like normally distributed data, and so it is inappropriate to use mean and standard deviation to estimate percentages for skewed data.



Problem 1 Analyzing Normally Distributed Data STEM

Zoology **The bar graph gives the weights of a population of female brown bears. The red curve shows how the weights are normally distributed about the mean, 115 kg. Approximately what percent of female brown bears weigh between 100 and 129 kg?**



Plan

How do you find this percent?
The percents for each bar are based on the same sample population of bears. You can add the percents.

Estimate and add the percents for the intervals 100–109, 110–119, and 120–129.

$23 + 42 + 23 = 88$

About 88% of female brown bears weigh between 100 and 129 kg.

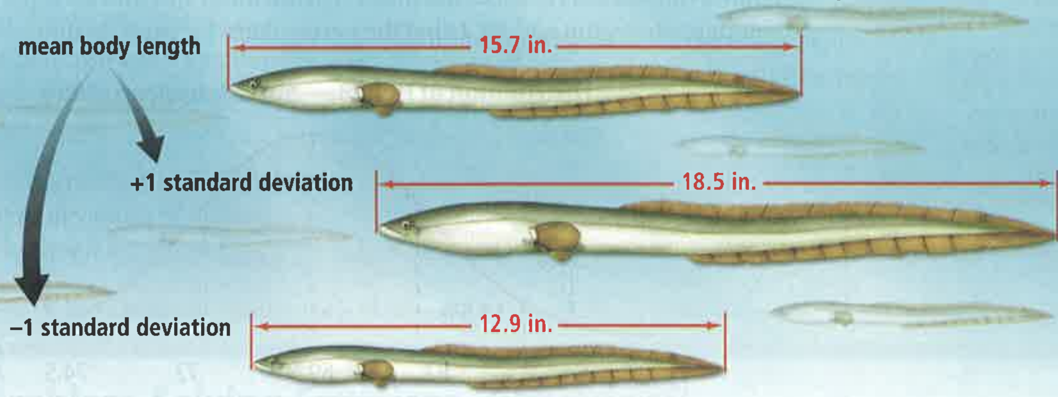
Got It? **1. a.** Approximately what percent of female brown bears in Problem 1 weigh less than 120 kg?

b. The standard deviation in the weights of female brown bears is about 10 kg. Approximately what percent of female brown bears have weights that are within 1.5 standard deviations of the mean?

When data are normally distributed, you can sketch the graph of the distribution using the fact that a normal curve has a symmetric bell shape.

Problem 2 Sketching a Normal Curve STEM

Zoology **For a population of male European eels, the mean body length and one positive and negative standard deviation is shown below. Sketch a normal curve showing the eel lengths at one, two, and three standard deviations from the mean.**



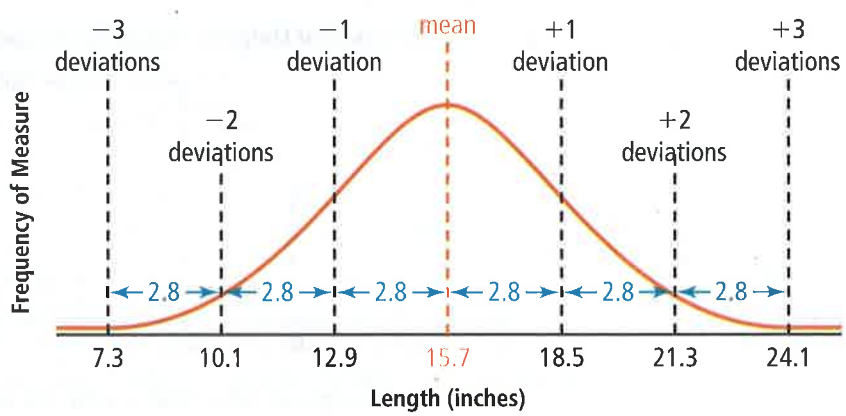
Know
The mean and the standard deviation of the population

Need
Lengths that are one, two, and three standard deviations from the mean

Plan
- Multiply the standard deviation by 1, 2, and 3.
- Draw vertical lines at the mean ± these values.
- Sketch the normal curve.

Think
How high do you draw the curve?
Unless you actually label the vertical scale, it doesn't matter.

Distribution of Body Lengths for Male European Eels



Got It? 2. For a population of female European eels, the mean body length is 21.1 in. The standard deviation is 4.7 in. Sketch a normal curve showing eel lengths at one, two, and three standard deviations from the mean.

When you show a probability distribution as a bar graph, the height of the bar indicates probability. For a normal distribution, however, the area between the curve and an interval on the x-axis represents probability.

Problem 3 Analyzing a Normal Distribution

Plan

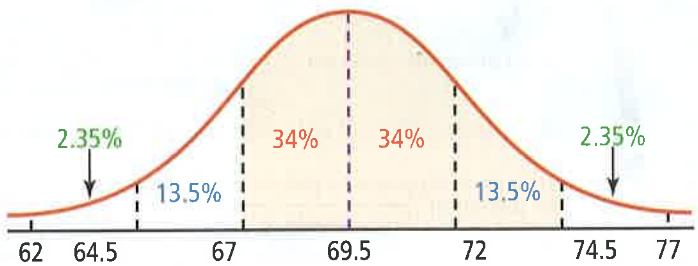
How do you divide the graph of the distribution?
Draw vertical lines at intervals that are one standard deviation wide, on both sides of the mean.

The heights of adult American males are approximately normally distributed with mean 69.5 in. and standard deviation 2.5 in.

A What percent of adult American males are between 67 in. and 74.5 in. tall?

Draw a normal curve. Label the mean. Divide the graph into sections of standard-deviation widths. Label the percentages for each section.

Distribution of Heights—Adult American Males



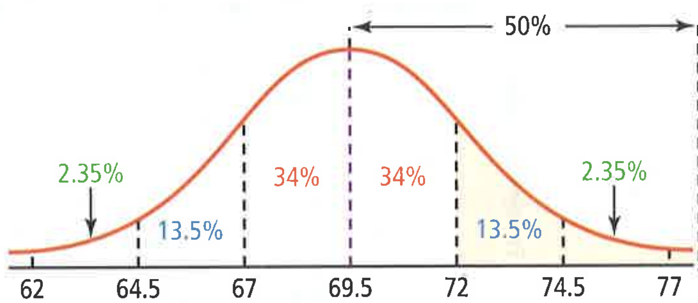
$$P(67 < \text{height} < 74.5) = 0.34 + 0.34 + 0.135 = 0.815$$

About 82% of adult American males are between 67 in. and 74.5 in. tall.

B In a group of 2000 adult American males, about how many would you expect to be taller than 6 ft (or 72 in.)?

Because the graph is symmetric about the mean, the right half of the distribution contains 50% of the data. If you subtract everything between 69.5 in. and 72 in. from the right half, only the part of the distribution that is greater than 72 in. remains.

Distribution of Heights—Adult American Males



$$P(\text{height} > 72) = 0.50 - 0.34 = 0.16$$

You would expect about 16% of the 2000 adult American males to be taller than 72 in. You would expect about $0.16 \cdot 2000 = 320$ to be over 6 ft tall.

Got It? **3.** The scores on the Algebra 2 final are approximately normally distributed with a mean of 150 and a standard deviation of 15.

a. What percentage of the students who took the test scored above 180?

b. If 250 students took the final exam, approximately how many scored above 135?

c. Reasoning If 13.6% of the students received a B on the final, how can you describe their scores? Explain.