# 11-1 Permutations and Combinations

**Objectives** To count permutations
To count combinations

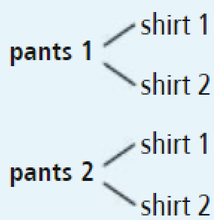
**take note**

### Key Concept Fundamental Counting Principle

If event $M$ can occur in $m$ ways and is followed by event $N$ that can occur in $n$ ways, then event $M$ followed by event $N$ can occur in $m \cdot n$ ways.

**Example** 3 pants and 2 shirts give $3 \cdot 2 = 6$ possible outfits.

**Here's Why It Works** Making a tree diagram, you can see that there are 3 groups of 2 outfits, or $3 \cdot 2 = 6$ outfits, starting with the pants.




You can extend the Fundamental Counting Principle to three or more events.

## Problem 1 Using the Fundamental Counting Principle

**Motor Vehicles** **The photos show Maryland license plates in 2004 and 1912. How many more 2004-style license plates were possible than 1912-style plates?**


For the 2004 license plates, there were places for three letters and three digits. Number of possible 2004 license plates:

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$


For the 1912 license plates, there were places for four digits. Number of possible 1912 license plates:

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$

$17{,}576{,}000 - 10{,}000 = 17{,}566{,}000$ Find the difference.

There were 17,566,000 more 2004-style license plates possible than 1912-style plates.

A **permutation** is an arrangement of items in a particular order. Suppose you wanted to find the number of ways to order three items. There are 3 ways to choose the first item, 2 ways to choose the second, and 1 way to choose the third. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 1 = 6$ permutations.

Using *factorial* notation, you can write $3 \cdot 2 \cdot 1$ as 3!, read "three factorial." For any positive integer $n$, ***n* factorial** is $n! = n(n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. Zero factorial is $0! = 1$.

## Problem 2 Finding the Number of Permutations of n Items

**In how many ways can you file 12 folders, one after another, in a drawer?**

Use the Fundamental Counting Principle to count the number of permutations of 12 items. There are 12 ways to select the first folder, 11 ways to select the next folder, and so on. The total number of permutations is

$$12! = 12 \cdot 11 \cdot \ldots \cdot 2 \cdot 1 = 479{,}001{,}600.$$

There are 479,001,600 ways to file 12 folders in a drawer.

take note

## Key Concept Number of Permutations

The number of permutations of $n$ items of a set arranged $r$ items at a time is

$${}_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \le r \le n.$$

**Example** ${}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 5040$

## Problem 3 Finding ${}_nP_r$

**Track** **Ten students are in a race. First, second, and third places will win medals. In how many ways can 10 runners finish first, second, and third (no ties allowed)?**

**Method 1** Use the Fundamental Counting Principle.

$$10 \cdot 9 \cdot 8 = 720$$

**Method 2** Use the permutation formula.

There are $n = 10$ runners to arrange taking $r = 3$ at a time.

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{Use the formula.}$$

$$= \frac{10!}{(10-3)!} \quad \text{Substitute 10 for } n \text{ and 3 for } r.$$

$$= \frac{10!}{7!} = 720 \quad \text{Simplify.}$$

There are 720 ways that 10 runners can finish in first, second, and third places.

take note

## Key Concept  Number of Combinations

The number of combinations of $n$ items of a set chosen $r$ items at a time is

$${}_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \leq r \leq n.$$

**Example** ${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \cdot 2!} = \frac{120}{6 \cdot 2} = 10$

## Problem 4 Finding ${}_nC_r$

**What is ${}_{13}C_4$, the number of combinations of 13 items taken 4 at a time?**

| Think | Write |
|---|---|
| You need the formula for number of combinations. Substitute 13 for $n$ and 4 for $r$. | ${}_nC_r = \frac{n!}{r!(n-r)!}$<br>${}_{13}C_4 = \frac{13!}{4!(13-4)!}$<br>$= \frac{13!}{4! \cdot 9!}$ |
| Write out the factorial in the numerator to make it easier to divide. Remove common factors. | $= \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 9!} = 715$ |

## Problem 5 Identifying Whether Order Is Important

**For each situation, determine whether you should use a permutation or combination. What is the answer to each question?**

**A** **A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawings to display. In how many different ways can he select the drawings?**

There is no reason why order is important. Use a combination.

$${}_nC_r = \frac{n!}{r!(n-r)!}; \qquad {}_8C_4 = \frac{8!}{4!(8-4)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{4!}} = 70$$

There are 70 ways to select the drawings.

**B** **You will draw winners from a total of 25 tickets in a raffle. The first ticket wins \$100. The second ticket wins \$50. The third ticket wins \$10. In how many different ways can you draw the three winning tickets?**

Values of the tickets depend on the order in which you draw them. Order is important. Use a permutation.

$${}_nP_r = \frac{n!}{(n-r)!}; \qquad {}_{25}P_3 = \frac{25!}{(25-3)!} = \frac{25!}{22!} = 25 \cdot 24 \cdot 23 = 13{,}800$$

There are 13,800 ways you can draw the winning tickets.