

1) Section 9.1 (6 points) Write the series represented by the summation notation given below. Then evaluate the sum.

$$\sum_{n=2}^6 \frac{3n}{2(n-1)}$$

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$$= \frac{6}{2(1)} + \frac{9}{2(2)} + \frac{12}{2(3)} + \frac{15}{2(4)} + \frac{18}{2(5)}$$

$$= 3 + \frac{9}{4} + 2 + \frac{15}{8} + \frac{9}{5}$$

$$= \frac{120 + 90 + 80 + 75 + 72}{40}$$

$$= \frac{437}{40}$$

2) Section 9.2 (5 points) Find the 12th term of the arithmetic sequence if the eighth term is 12 and the 23rd term is 50.

$$a_{23} = a_8 + 15d$$

$$50 = 12 + 15d$$

$$38 = 15d$$

$$d = \frac{38}{15}$$

$$a_{12} = a_8 + 4d$$

$$a_{12} = 12 + 4\left(\frac{38}{15}\right)$$

$$= 12 + \frac{152}{15}$$

$$= \frac{180}{15} + \frac{152}{15}$$

$$a_{12} = \frac{332}{15}$$

3) Section 9.4 (5 points) Find the sum of the first 25 terms of the arithmetic sequence given below.

3, 7, 11, 15, ...

$$\underbrace{3 + 7 + 11 + 15 + \dots + a_{25}}_{25 \text{ Terms}}$$

$$Sum = \frac{n(a_1 + a_n)}{2}$$

$$\begin{aligned} a_{25} &= a_1 + 24d \\ &= 3 + 24(4) \\ &= 3 + 96 \\ &= 99 \end{aligned}$$

$$\begin{aligned} Sum &= \\ &= \frac{25(3 + 99)}{2} \\ &= \frac{25(102)}{2} \\ &= 25(51) \\ &= 1275 \end{aligned}$$

4) Section 9.4 (5 points) Find the sum of the arithmetic series given below.

$$\frac{1}{3} + \frac{2}{3} + 1 + \frac{4}{3} + \dots + 22$$

$$\text{Sum} = \frac{n(a_1 + a_n)}{2} = \frac{n\left(\frac{1}{3} + 22\right)}{2}$$

$$a_n = a_1 + (n-1)d$$

$$22 = \frac{1}{3} + (n-1)\left(\frac{1}{3}\right)$$

$$22 = \cancel{\frac{1}{3}} + \frac{1}{3}n - \cancel{\frac{1}{3}}$$

$$22 = \frac{1}{3}n$$

$$n = 66$$

$$= \frac{66\left(\frac{1}{3} + 22\right)}{2}$$

$$= 33\left(\frac{67}{3}\right)$$

$$= 11(67)$$

$$= 737$$

5) Section 9.1/9.4 (5 points) Find the sum of the first 5 terms of the recursive sequence given by  $a_n = 2a_{n-1} + 3$  with  $a_1 = -2$ .

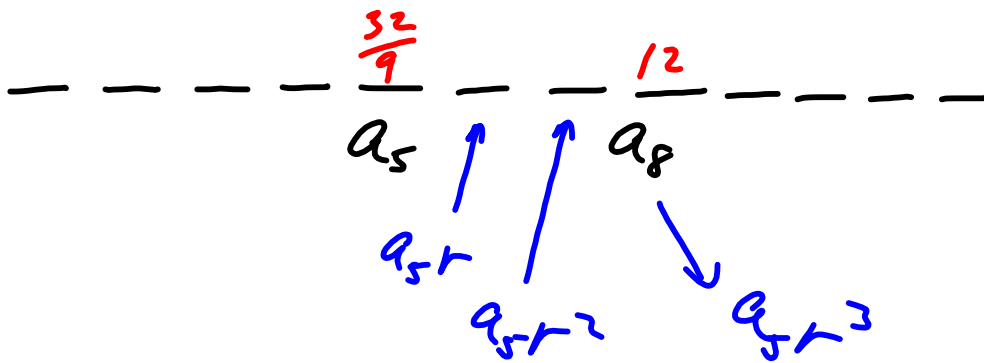
$$-2, -1, 1, 5, 13$$

$$-2 + (-1) + 1 + 5 + 13$$

$$= 16$$

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6) Section 9.3 (5 points) Find the 12th term of the geometric sequence if the eighth term is 12 and the fifth term is  $\frac{32}{9}$ .



$$a_8 = a_5 r^3$$

$$12 = \frac{32}{9} r^3$$

$$r^3 = 12 \left( \frac{9}{32} \right) = 3 \left( \frac{9}{8} \right) = \frac{27}{8}$$

$$r^3 = \frac{27}{8}$$

$$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

$$a_{12} = a_8 r^4 = 12 \left( \frac{3}{2} \right)^4$$

$$= 12 \left( \frac{81}{16} \right) = 3 \left( \frac{81}{4} \right)$$

$$a_{12} = \frac{243}{4}$$

7) **Section 9.5** (5 points) Evaluate the sum of the finite Geometric Series given below.

$$\begin{aligned} \text{Sum} &= \frac{a_1 (1-r^n)}{1-r} \\ &= \frac{3(1-4^6)}{1-4} \\ &= -(1-4^6) \\ &= 4^6 - 1 \\ &= 4095 \end{aligned}$$

8) **Section 9.5** (5 points) Find the sum of the infinite Geometric Series given below.

$$\sum_{n=1}^{\infty} -\frac{1}{6} \left(-\frac{1}{2}\right)^{n-1}$$

This series may also be represented as

$$-\frac{1}{6} + \frac{1}{12} - \frac{1}{24} + \frac{1}{48} - \frac{1}{96} + \dots$$

$$\begin{aligned} \text{Sum} &= \frac{a_1}{1-r} = \frac{-\frac{1}{6}}{1 - \left(-\frac{1}{2}\right)} = \frac{-\frac{1}{6}}{\frac{3}{2}} \\ &= -\frac{1}{6} \left(\frac{2}{3}\right) = \boxed{-\frac{1}{9}} \end{aligned}$$



9) Section 9.5 (5 points) Find the sum of the 11 terms in the geometric series given below.

$$-\frac{8}{3} + \frac{4}{3} - \frac{2}{3} + \frac{1}{3} - \frac{1}{6} + \frac{1}{12} - \dots - \frac{1}{384}$$

$$Sum = \frac{a_1(1-r^n)}{1-r}$$

$$= \frac{-\frac{8}{3} \left(1 - \left(-\frac{1}{2}\right)^{11}\right)}{1 - \left(-\frac{1}{2}\right)}$$

$$= \frac{-\frac{8}{3} \left(1 - \left(-\frac{1}{2}\right)^{11}\right)}{\frac{3}{2}}$$

$$= -\frac{16}{9} \left(1 - \left(-\frac{1}{2}\right)^{11}\right)$$

10) Section 3.1 (6 points) Solve the following system of equations by using the **Graphing Method**.

$$3x - 4y = 24$$

$$3x + 2y = 6$$

$$\underline{3x - 4y = 24}$$

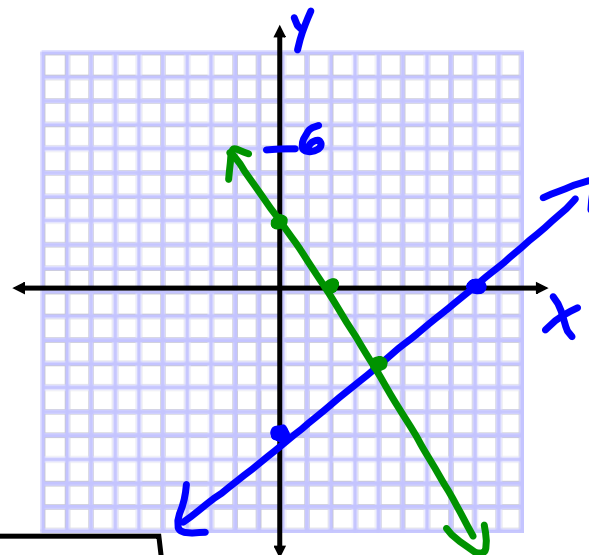
$$X_{INT}: 8$$

$$Y_{INT}: -6$$

$$\underline{3x + 2y = 6}$$

$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3$$



$$(4, -3)$$

12) Section 3.2 (6 points) Solve the following system of equations by using the **Method of Elimination**.

$$3x - 2y = 22$$

$$-5x + 6y = -36$$

$$\begin{array}{r} 9x - 6y = 66 \\ -5x + 6y = -36 \\ \hline \end{array}$$

$$4x = 30$$

$$x = \frac{15}{2}$$

$$\begin{array}{r} 15x - 10y = 110 \\ -15x + 18y = -108 \\ \hline \end{array}$$

$$8y = 2$$

$$y = \frac{1}{4}$$

$$\left( \frac{15}{2}, \frac{1}{4} \right)$$

11) Section 3.2 (6 points) Solve the following system of equations by using the Method of Substitution.

$$6x - 8y = 6$$

$$-3x + 2y = -2$$

$$-3x + 2y = -2$$

$$\frac{2y}{2} = \frac{3x}{2} - \frac{2}{2}$$

$$y = \frac{3}{2}x - 1$$

$$6x - 8\left(\frac{3}{2}x - 1\right) = 6$$

$$6x - 12x + 8 = 6$$

$$-6x = -2$$

$$x = \frac{1}{3}$$

$$y = \frac{3}{2}x - 1 = \frac{3}{2}\left(\frac{1}{3}\right) - 1$$

$$= \frac{1}{2} - 1$$

$$= -\frac{1}{2}$$

$$\left(\frac{1}{3}, -\frac{1}{2}\right)$$

13) Section 3.5 (8 points) Solve the following system of equations.

$$\begin{array}{r}
 \textcircled{1} \quad 3x + 3y + 5z = 1 \\
 \textcircled{2} \quad 3x + 5y + 9z = 0 \\
 \textcircled{3} \quad 5x + 9y + 17z = 0
 \end{array}$$

$$\begin{array}{r}
 -1 \textcircled{1} \quad -3x - 3y - 5z = -1 \\
 + \textcircled{2} \quad \quad 3x + 5y + 9z = 0 \\
 \hline
 \quad \quad 2y + 4z = -1
 \end{array}$$

$$\begin{array}{r}
 -5 \textcircled{1} \quad -15x - 15y - 25z = -5 \\
 + 3 \textcircled{3} \quad \quad 15x + 27y + 51z = 0 \\
 \hline
 \quad \quad 12y + 26z = -5
 \end{array}$$

NOTE: MAKE SURE THE SAME VARIABLE HAS BEEN ELIMINATED.

$$\begin{array}{r}
 2y + 4z = -1 \\
 12y + 26z = -5 \\
 \rightarrow \quad \quad \quad \begin{array}{r}
 -12y - 24z = 6 \\
 \underline{12y + 26z = -5} \\
 2z = 1
 \end{array}
 \end{array}$$

$$z = \frac{1}{2}$$

$$2y + 4\left(\frac{1}{2}\right) = -1$$

$$2y + 2 = -1$$

$$2y = -3$$

$$y = -\frac{3}{2}$$

FROM ORIGINAL

$$3x + 3y + 5z = 1$$

$$3x + 3\left(-\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 1$$

$$3x - \frac{9}{2} + \frac{5}{2} = 1$$

$$3x - 2 = 1$$

$$3x = 3$$

$$x = 1$$

$$\left(1, -\frac{3}{2}, \frac{1}{2}\right)$$