

Algebra II Review Sheet Chapter 8 Rational Functions.notebook

1) (10 points) Sketch the graph of the function below. Identify all asymptotes and all intercepts. State the domain and range of this function.

$$f(x) = \frac{2x^2 - 18}{x^2 - 4}$$

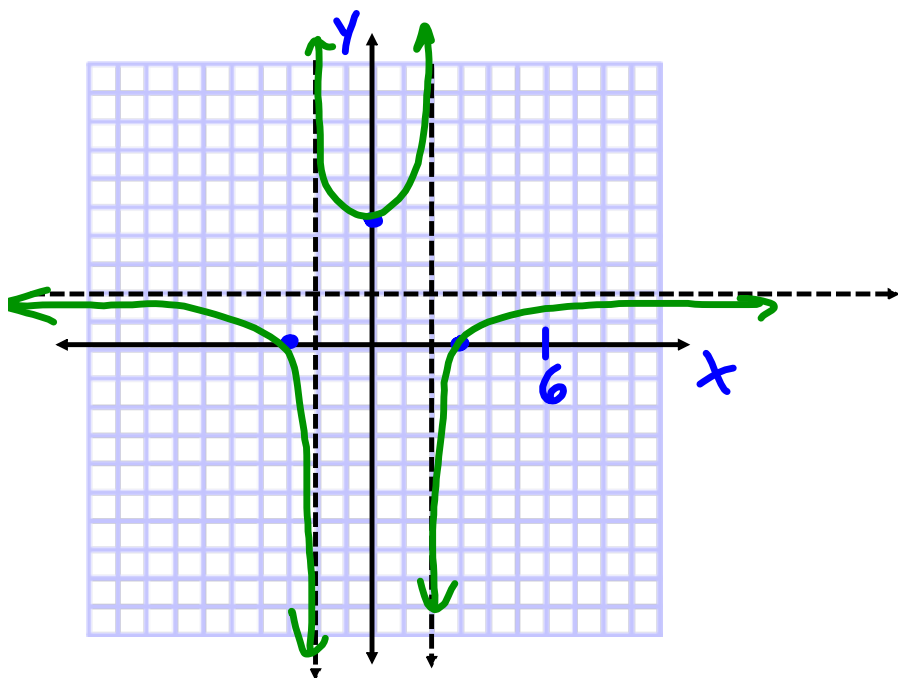
$$f(x) = \frac{2(x^2 - 9)}{x^2 - 4} = \frac{2(x-3)(x+3)}{(x-2)(x+2)}$$

y-intercept: $\frac{-18}{-4} = \frac{9}{2}$

x-intercept: $3, -3$

Vertical Asymptotes: $x = 2, x = -2$

Horizontal Asymptote: $y = \frac{2}{1} = 2$



DOMAIN: $x \neq 2, -2$

RANGE: $(-\infty, 2) \cup [\frac{9}{2}, \infty)$

2) (6 points) Perform the operations on the rational expressions and simplify.

$$\frac{x^2 - 5x - 14}{x^3 - 6x^2 - 7x} \cdot (x^2 - 4x - 5) \div \frac{x^2 + x - 30}{2x}$$

$$\frac{(x-7)(x+2)}{x(x^2-6x-7)} \cdot \cancel{(x-5)(x+1)} \cdot \frac{2x}{\cancel{(x+6)(x-5)}}$$

$$\frac{\cancel{(x-7)}(x+2)}{\cancel{x}(\cancel{x-7})(x+1)} \cdot (x+1) \cdot \frac{\cancel{2x}}{x+6}$$

$$= \frac{(x+2)\cancel{(x+1)}(2)}{\cancel{(x+1)}(x+6)} = \frac{2(x+2)}{x+6}$$

3) (8 points) Solve the rational equation below.

$$\frac{10x^2 + x - 2}{2x^2 - 9x - 18} = \frac{4x}{x-6} + \frac{2x-3}{2x+3}$$

$$\frac{10x^2 + x - 2}{(x-6)(2x+3)} = \frac{4x}{x-6} + \frac{2x-3}{2x+3}$$

$$\frac{10x^2 + x - 2}{\cancel{(x-6)(2x+3)}} \cdot \frac{\cancel{(x-6)(2x+3)}}{\cancel{(x-6)(2x+3)}} = \frac{4x}{\cancel{x-6}} \cdot \frac{\cancel{(x-6)(2x+3)}}{\cancel{(x-6)(2x+3)}} + \frac{2x-3}{\cancel{2x+3}} \cdot \frac{\cancel{(x-6)(2x+3)}}{\cancel{(x-6)(2x+3)}}$$

$$10x^2 + x - 2 = 4x(2x+3) + (2x-3)(x-6)$$

$$\cancel{10x^2} + x - 2 = \cancel{8x^2} + \cancel{12x} + \cancel{2x^2} - \cancel{12x} - 3x + 18$$

$$x - 2 = -3x + 18$$

$$4x = 20$$

$$x = 5$$

CHECK IT!

4) (6 points) Perform the operations and simplify.

$$\frac{x}{x^2 + 2x + 1} + \frac{1}{x^2 + 5x + 4}$$

$$\frac{x}{(x+1)(x+1)} + \frac{1}{(x+1)(x+4)}$$

$$= \frac{x}{(x+1)(x+1)} \cdot \frac{x+4}{x+4} + \frac{1}{(x+1)(x+4)} \cdot \frac{x+1}{x+1}$$

$$= \frac{x(x+4) + 1(x+1)}{(x+1)^2(x+4)}$$

$$= \frac{x^2 + 4x + x + 1}{(x+1)^2(x+4)}$$

$$= \frac{x^2 + 5x + 1}{(x+1)^2(x+4)}$$

Algebra II Review Sheet Chapter 8 Rational Functions.notebook

5) (6 points) Simplify the complex fraction.

$$\frac{\frac{x}{x-3} - \frac{2}{3}}{\frac{10}{3x} + \frac{x^2}{x-3}}$$

$$\begin{aligned} & \frac{3x(x-3) \left(\frac{x}{x-3} - \frac{2}{3} \right)}{3x(x-3) \left(\frac{10}{3x} + \frac{x^2}{x-3} \right)} \\ &= \frac{3x^2 - 2x(x-3)}{10(x-3) + 3x^3} \\ &= \frac{3x^2 - 2x^2 + 6x}{10x - 30 + 3x^3} \\ &= \frac{x^2 + 6x}{3x^3 + 10x - 30} \end{aligned}$$

OR

$$\begin{aligned} & \frac{\frac{x}{x-3} - \frac{2}{3}}{\frac{10}{3x} + \frac{x^2}{x-3}} \\ &= \frac{\frac{x}{x-3} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{x-3}{x-3}}{\frac{10}{3x} \cdot \frac{x-3}{x-3} + \frac{x^2}{x-3} \cdot \frac{3x}{3x}} \\ &= \frac{3x - 2(x-3)}{3(x-3)} \cdot \frac{10(x-3) + x^2(3x)}{3x(x-3)} \\ &= \frac{3x - 2x + 6}{3(x-3)} \cdot \frac{10x - 30 + 3x^3}{3x(x-3)} \\ &= \frac{x + 6}{3(x-3)} \cdot \frac{3x^3 + 10x - 30}{3x(x-3)} \\ &= \frac{x + 6}{\cancel{3(x-3)}} \cdot \frac{\cancel{3x(x-3)}}{3x^3 + 10x - 30} \\ &= \frac{x(x+6)}{3x^3 + 10x - 30} \end{aligned}$$

Algebra II Review Sheet Chapter 8 Rational Functions.notebook

6) (10 points) Sketch the graph of the function below. Identify all asymptotes and all intercepts. State the domain and range of this function.

$$f(x) = \frac{(2x-6)(x^2+3x-10)}{(x-2)(x^2+x-2)}$$

$$f(x) = \frac{2(x-3)(x+5)\cancel{(x-2)}}{\cancel{(x-2)}(x-1)(x+2)}$$

HOLE AT $x=2$

$$f(x) = \frac{2(x-3)(x+5)}{(x-1)(x+2)}$$

y-intercept: $\frac{60}{4} = 15$

x-intercept: $3, -5$

Vertical Asymptotes: $x=1, x=-2$

Horizontal Asymptote: $y=2$

