1) **Section 14.1** (5 points) Simplify the expression given below.

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{1+\sin\theta}\cdot\frac{\cos\theta}{\cos\theta} + \frac{1+\sin\theta}{\cos\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{1 + 1 + 2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = \boxed{2\sec\theta}$$

2) **Section 14.1** (5 points) Prove the identity given below.

$$\frac{1-\sin^2\theta}{1+\cot^2\theta} = \sin^2\theta\cos^2\theta$$

Recall:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$$\frac{1-\sin^2\theta}{1+\cot^2\theta} = \frac{\cos^2\theta}{\csc^2\theta} = \sin^2\theta\cos^2\theta$$

3) **Section 14.2** (6 points) Solve the equation below.

$$2\cos^2 x - \cos x = 2 - \sec x$$

$$2\cos^2 x - \cos x = 2 - \frac{1}{\cos x}$$

MULTIPLY BY $\cos x$

$$2\cos^3 x - \cos^2 x = 2\cos x - 1$$

$$2\cos^3 x - \cos^2 x - 2\cos x + 1 = 0$$

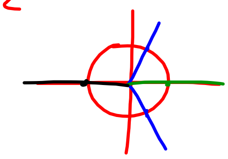
$$(2\cos^3 x - \cos^2 x) - (2\cos x - 1) = 0$$

$$\cos^2 x(2\cos x - 1) - 1(2\cos x - 1) = 0$$

$$(2\cos x - 1)(\cos^2 x - 1) = 0$$

$$(2\cos x - 1)(\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = 1$ $\qquad$ $\cos x = -1$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = 0$ $\qquad$ $x = \pi$

$$\boxed{\begin{array}{l} x = 0 + n\pi \\ x = \frac{\pi}{3} + 2n\pi \\ x = \frac{5\pi}{3} + 2n\pi \end{array}}$$

4) **Section 14.2** (6 points) Determine all solutions to the equation given below. Express your answers in radian measure.

$$2\cos^2 x - \sin x - 1 = 0$$

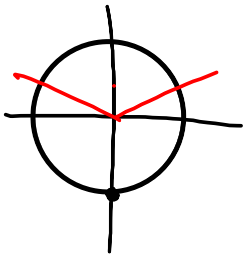
$$2(1 - \sin^2 x) - \sin x - 1 = 0$$

$$2 - 2\sin^2 x - \sin x - 1 = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

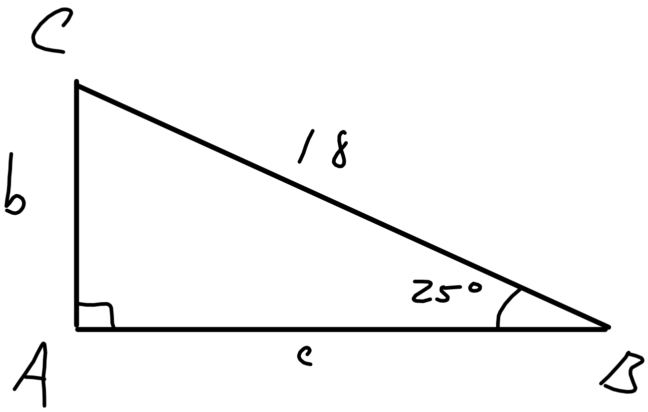


$\sin x = \frac{1}{2}$ $\qquad\qquad$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad\qquad$ $x = \frac{3\pi}{2}$

$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + 2n\pi \\ x &= \frac{5\pi}{6} + 2n\pi \\ x &= \frac{3\pi}{2} + 2n\pi \end{aligned}}$$

5) **Section 14.3** (4 points) In $\triangle ABC$, $\angle A = 90^\circ$, $\angle B = 25^\circ$, and $a = 18$. Find b and c.



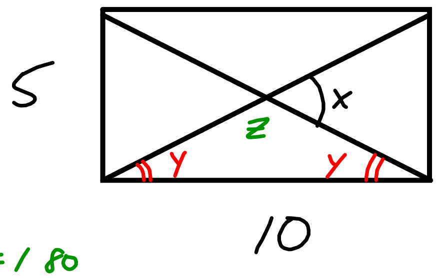
$$\sin 25^\circ = \frac{b}{18}$$

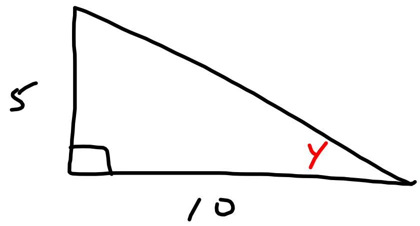
$$b = 18 \sin 25^\circ$$

$$\cos 25^\circ = \frac{c}{18}$$

$$c = 18 \cos 25^\circ$$

6) **Section 14.3** (4 points) A rectangle is 5in high and 10in long. Find the measure of the acute angle between its diagonals.





$z + x = 180$

$z + 2y = 180$

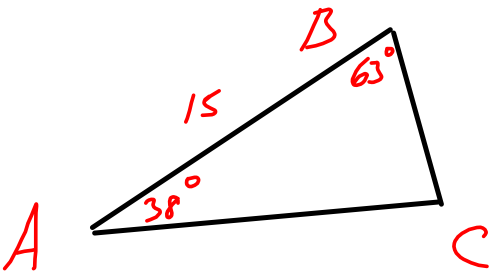
$x = 2y$

$\tan y = \frac{5}{10} = \frac{1}{2}$

$y = \tan^{-1}\left(\frac{1}{2}\right)$

$$\boxed{x = 2\tan^{-1}\left(\frac{1}{2}\right)}$$

7) **Section 14.4** (5 points) Solve the triangle $\triangle ABC$ if $\angle A = 38°$, $\angle B = 63°$, and $c = 15$.



$C = 180 - 38 - 63$

$\boxed{C = 79°}$

$A = 38°$ $a =$ SEE BELOW

$B = 63°$ $b =$

$C = 79°$ $c = 15$

$$\frac{\sin 79}{15} = \frac{\sin 38}{a}$$

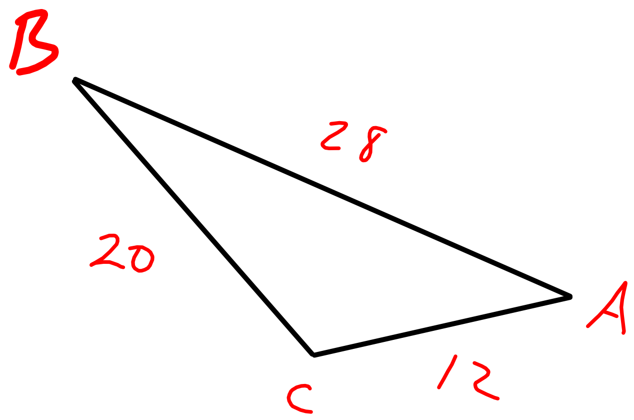
$$a \sin 79 = 15 \sin 38$$

$$\boxed{a = \frac{15 \sin 38}{\sin 79}}$$

$$\frac{\sin 79}{15} = \frac{\sin 63}{b}$$

$$\boxed{b = \frac{15 \sin 63}{\sin 79}}$$

8) **Section 14.5** (5 points) Solve the triangle $\triangle ABC$ if $a = 20$, $b = 12$, and $c = 28$.



$$\boxed{\begin{array}{ll} A \approx 38.2^\circ & a = 20 \\ B \approx 21.8^\circ & b = 12 \\ C \approx 120 & c = 28 \end{array}}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$20^2 = 12^2 + 28^2 - 2(12)(28) \cos A$$

$$400 = 144 + 784 - 672 \cos A$$

$$400 = 928 - 672 \cos A$$

$$-528 = -672 \cos A$$

$$\cos A = \frac{-528}{-672} = \frac{132}{168} = \frac{33}{42} = \frac{11}{14}$$

$$A = \cos^{-1}\left(\frac{11}{14}\right) \approx 38.2^\circ$$

$$\frac{\sin 38.2}{20} = \frac{\sin B}{12}$$

$$\sin B = \frac{12 \sin 38.2}{20}$$

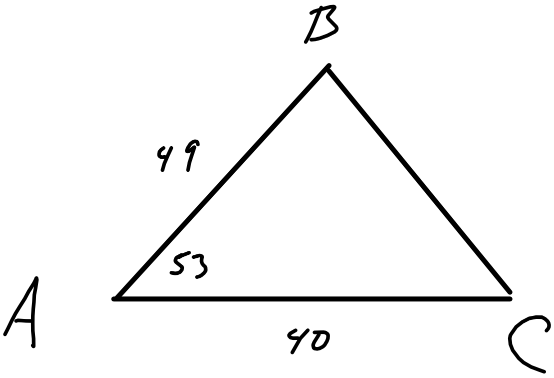
$$\sin B \approx .3711$$

$$B \approx 21.8$$

$$C = 180 - 38.2 - 21.8$$

$$= 120$$

9) **Section 14.5** (5 points) Solve the triangle $\triangle ABC$ if $\angle A = 53°$, $b = 40$, and $c = 49$.



$A = 53$ $a \approx 40.5$

$B \approx 52.0°$ $b = 40$

$C \approx 75.0°$ $c = 49$

$C = 180 - 52.0 - 53$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 40^2 + 49^2 - 2(40)(49)\cos 53$$

$$a^2 = 1600 + 2401 - 3920 \cos 53$$

$$a^2 = 4001 - 3920 \cos 53$$

$$a = \sqrt{4001 - 3920 \cos 53}$$

$$a \approx 40.5$$

$$\frac{\sin 53}{40.5} = \frac{\sin B}{40}$$

$$\sin B = \frac{40 \sin 53}{40.5}$$

$$B \approx 52.0°$$

11) **Section 14.6** (5 points) Find $\cos(\alpha - \theta)$ if the following conditions are true.

$\sin\alpha = \frac{12}{13}$ where $\frac{\pi}{2} < \alpha < \pi$

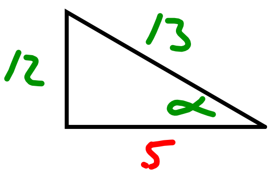
$\cos\theta = \frac{7}{25}$ where $-2\pi < \theta < -\frac{3\pi}{2}$

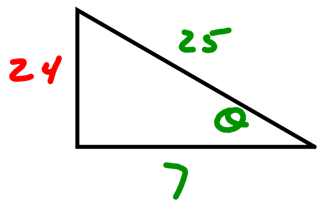
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\cos(\alpha - \theta) = \cos\alpha\cos\theta + \sin\alpha\sin\theta$$

$$= \cos\alpha\left(\frac{7}{25}\right) + \left(\frac{12}{13}\right)\sin\theta$$

STILL NEED $\cos\alpha$ AND $\sin\theta$



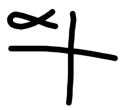


$x^2 + 12^2 = 13^2$

So, $\cos\alpha = -\frac{5}{13}$

NEGATIVE SINCE

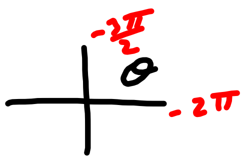
$\frac{\pi}{2} < \alpha < \pi$



$7^2 + y^2 = 25^2$

So $\sin\theta = \frac{24}{25}$

POSITIVE SINCE



$$\cos(\alpha - \theta) = \cos\alpha\left(\frac{7}{25}\right) + \left(\frac{12}{13}\right)\sin\theta$$

$$= -\frac{5}{13}\left(\frac{7}{25}\right) + \left(\frac{12}{13}\right)\left(\frac{24}{25}\right)$$

$$= \frac{-35 + 288}{325}$$

$$= \frac{253}{325}$$

12) **Section 14.6** (4 points) Use the formula for $\sin(x-y)$ to find the exact value of $\sin\frac{\pi}{12}$.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\sin\frac{\pi}{12} = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

13) **Section 14.7** (4 points) Use a half-angle formula to evaluate $\cos \frac{\pi}{8}$.

$$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

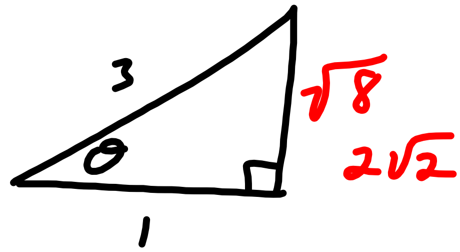
$$\cos\frac{\pi}{8} = \cos\left(\frac{\frac{\pi}{4}}{2}\right) = \pm\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$$

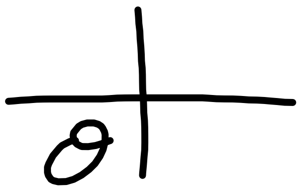
$$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

As $\frac{\pi}{8}$ is in QI, use +

14) **Section 14.7** (4 points) Find $\tan(2\theta)$ if $\cos\theta = -\frac{1}{3}$ and where $180^\circ < \theta < 270^\circ$.

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$





So, $\tan\theta > 0$

$$1^2 + x^2 = 3^2$$

$$x = 2\sqrt{2}$$

$$\tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{2\sqrt{2}}{1}$$

$$= 2\sqrt{2}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2(2\sqrt{2})}{1-(2\sqrt{2})^2} = \frac{4\sqrt{2}}{1-8}$$

$$\boxed{= \frac{-4\sqrt{2}}{7}}$$